Workshop on Many-Body Quantum Magic
MBQM2024

Abu Dhabi, November 18-20, 2024

Technology Innovation Institute

Entanglement and Magic in Quantum Systems: From Few-Body Physics to Many-Body Collectivity

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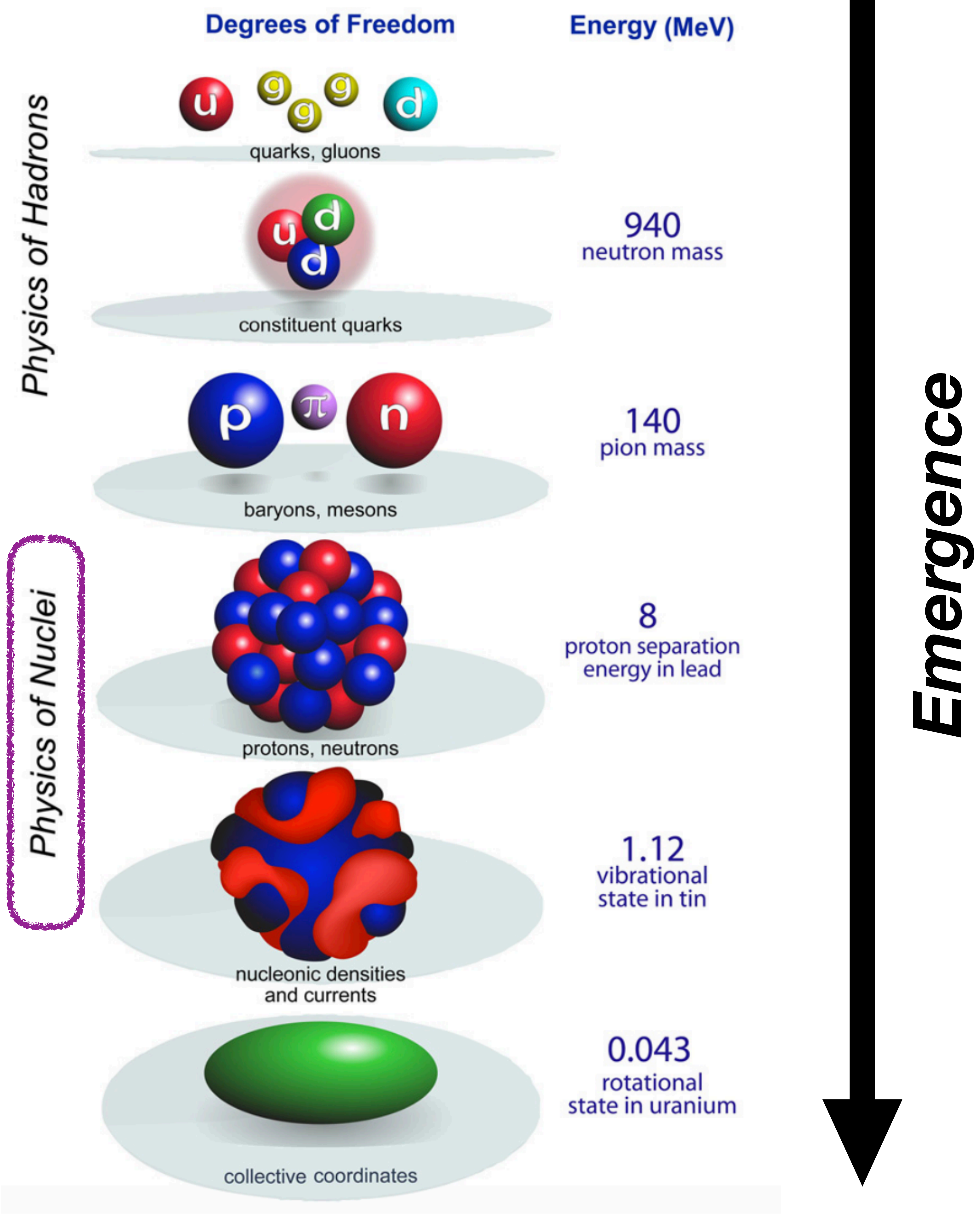
Atomic Nuclei as unique quantum many-body systems

- two-specie mesoscopic systems composed of 2 to ~300 protons and neutrons interacting via strong many-body nuclear forces

- various phenomena: single-particle shells, clustering, halos, emergent collective phenomena: superfluidity, vibrations, rotations…

Nuclei are great laboratories to learn about how matter organizes, how collective phenomena emerge from fundamental constituents and about properties of QMB systems in general.

Scales of nuclear physics



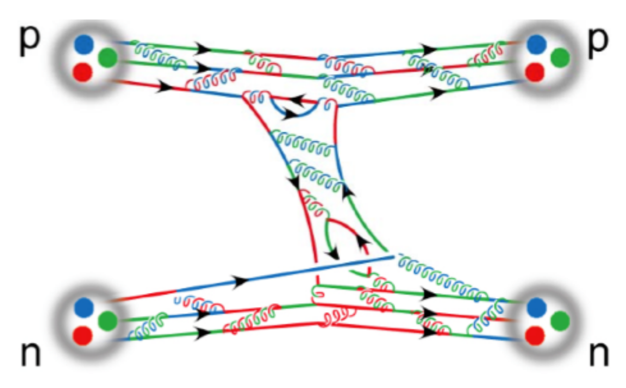
Bertsch, Dean, Nazarewicz SciDAC review (2007)

Towards Quantum Simulations of QMB Systems

Inspired by "Quantum simulation of fundamental particles and forces", Bauer, Davoudi, Klco, Savage, Nature Rev. Phy. 5, 420 (2023)

Motivational Questions

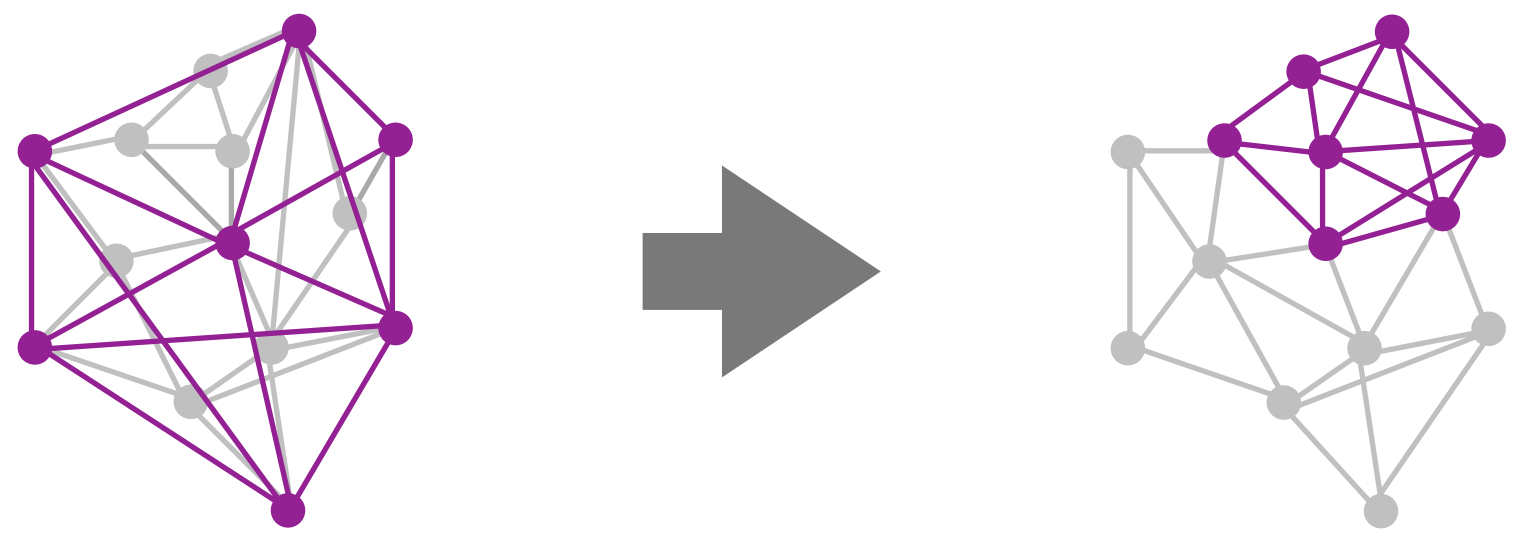
- What is the role played by entanglement and magic in the structure and dynamics of nuclear and other QMB systems? What are possible connections with underlying forces and symmetries?



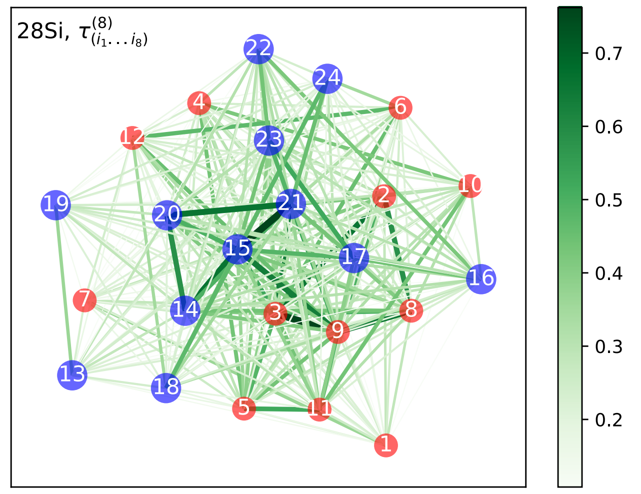
e.g. "Entanglement Suppression and Emergent Symmetries of Strong Interactions" Beane, Kaplan, Klco, Savage, PRL122,102001 (2019).

"Entanglement minimization in hadronic scattering with pions" Beane, Farrell, Varma. Int. J. Mod. Phys. A 36,2150205 (2021).

- In turn, can these concepts guide the development of new formulations of (nuclear) QMB problems, and of improved algorithms for hybrid classical/quantum simulations?

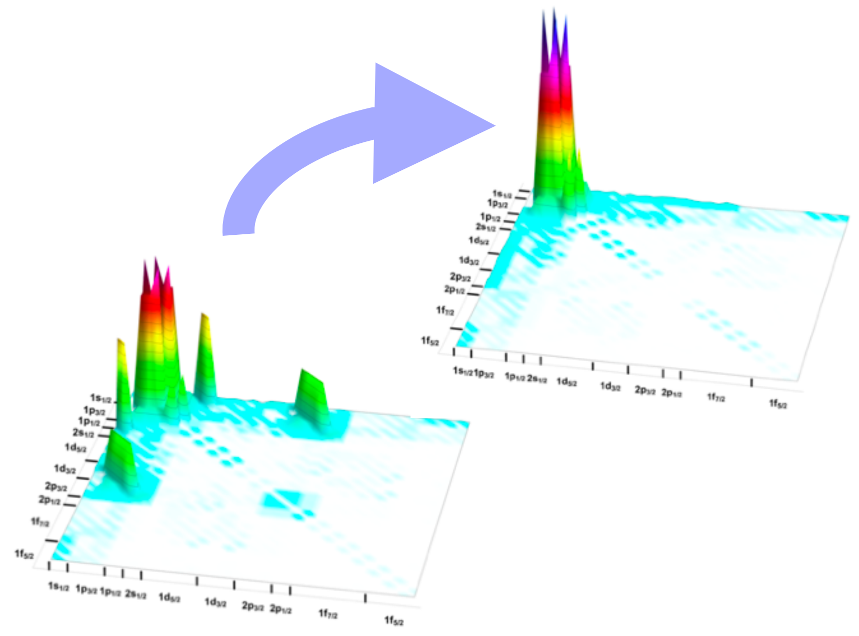


Outline



★ Multi-Partite Entanglement and Magic in the structure Nuclei

Brökemeier, Hengstenberg, Keeble, CR, Rocco & Savage, arXiv:2409.12064

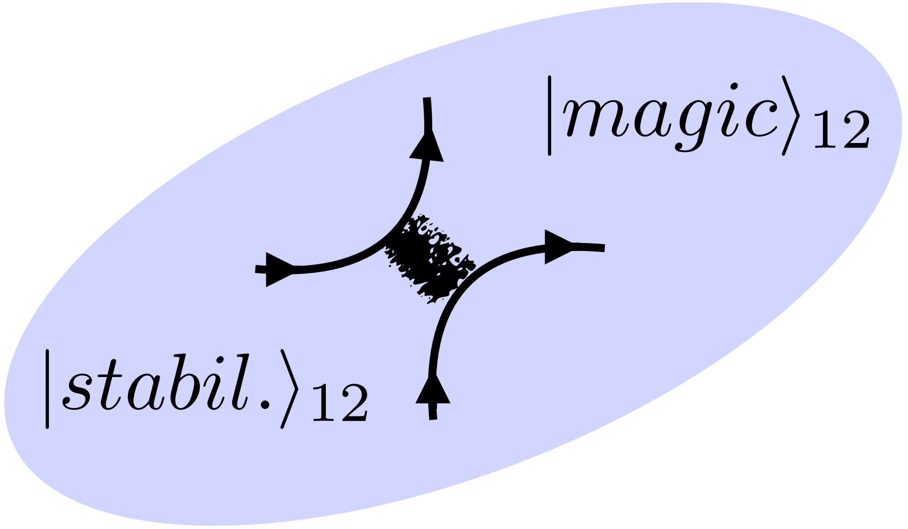


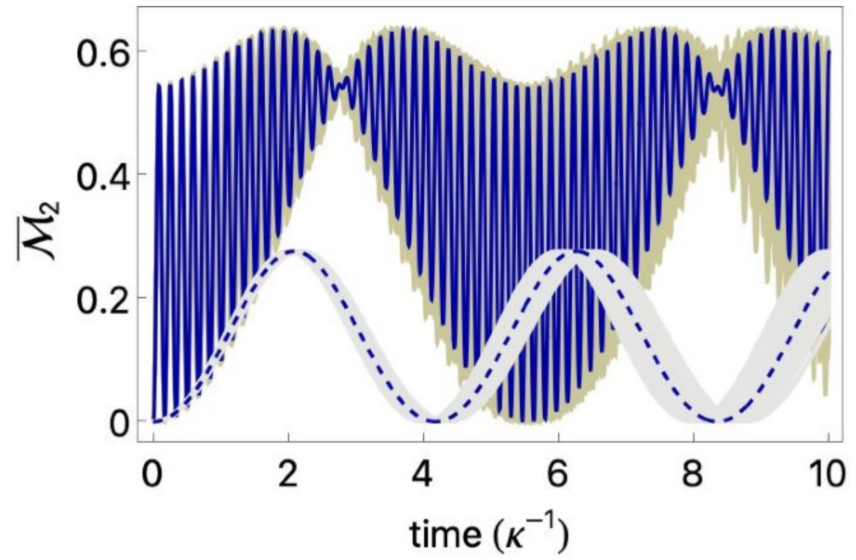
★ Entanglement and Magic Rearrangement for more efficient quantum simulations of QMB systems

CR, Savage, Pillet, PRC 103, 034325 (2021); CR & Savage PRC 108, 024313 (2023); Hengstenberg, CR, Savage EPJA 59, 231 (2023)

★ The Magic Power in Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268

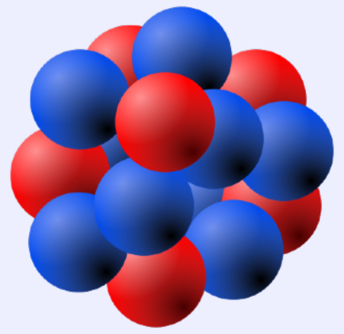




★ Entanglement and Magic in 3-flavour Neutrino dynamics mapped onto qutrits

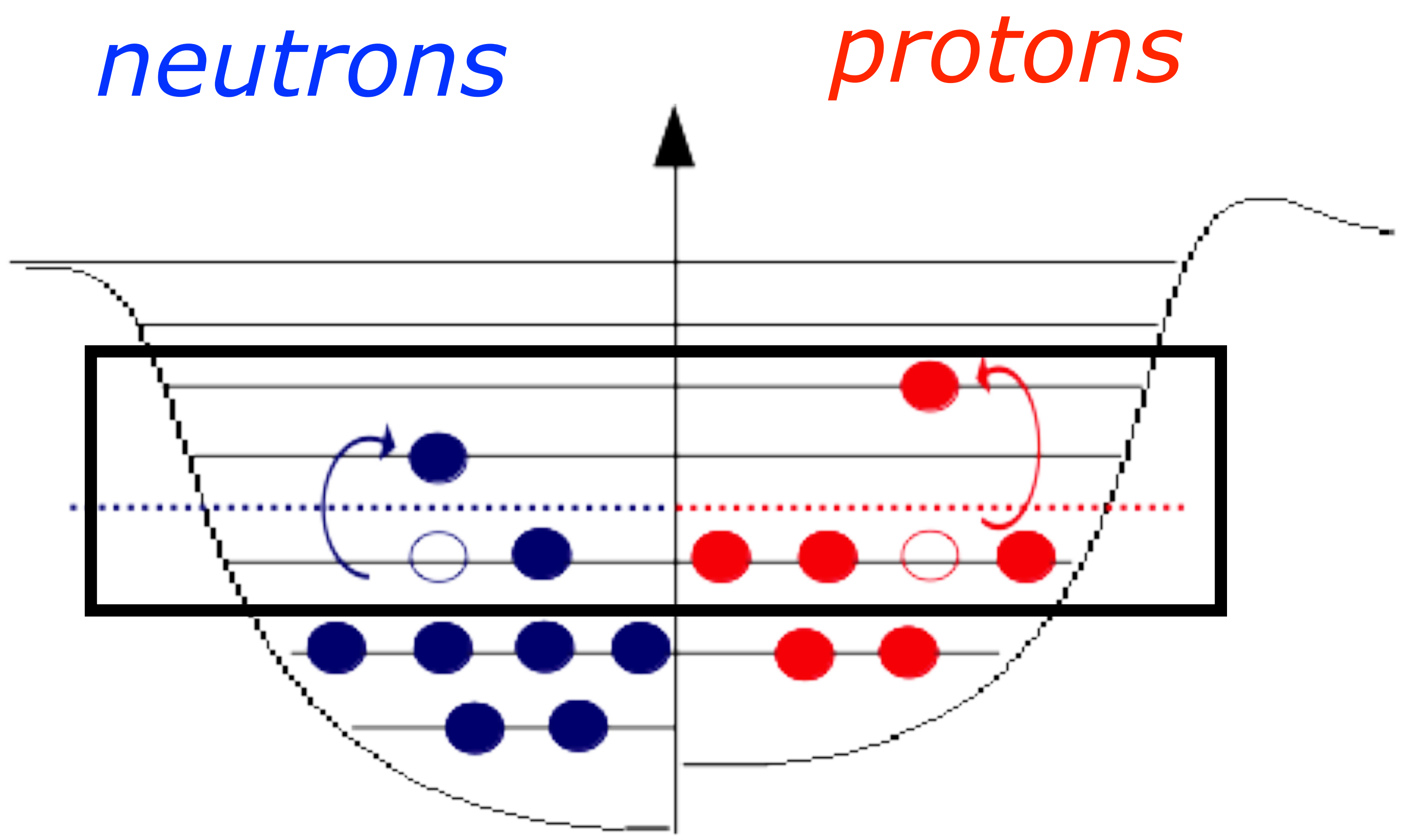
Chernyshev, CR, Savage arXiv:2411.04203

Entanglement and Magic in Nuclei



$$|\Psi\rangle = \sum_{\pi\nu} C_{\pi\nu} |\phi_\pi\rangle \otimes |\phi_\nu\rangle$$

$$= \sum_{\substack{n_{\pi_1} n_{\pi_2} \ldots n_{\nu_1} n_{\nu_2} \ldots \\ n_i = 0,1}} C_{n_{\pi_1} n_{\pi_2} \ldots n_{\nu_1} n_{\nu_2} \ldots} |n_{\pi_1} n_{\pi_2} \ldots n_{\nu_1} n_{\nu_2} \ldots\rangle$$



active-space calculations

Entanglement and Magic in Nuclei

Multi-Partite entanglement via n-tangles

Wong, Christensen, PRA 63, 044301 (2001)

$$\tau^{(n)}_{(i_1 \ldots i_n)} = \left| \langle \Psi | \hat{\sigma_y}^{(i_1)} \otimes \ldots \otimes \hat{\sigma_y}^{(i_n)} | \Psi^* \rangle \right|^2$$

⇒ n-tangles related to n/2-body entanglement

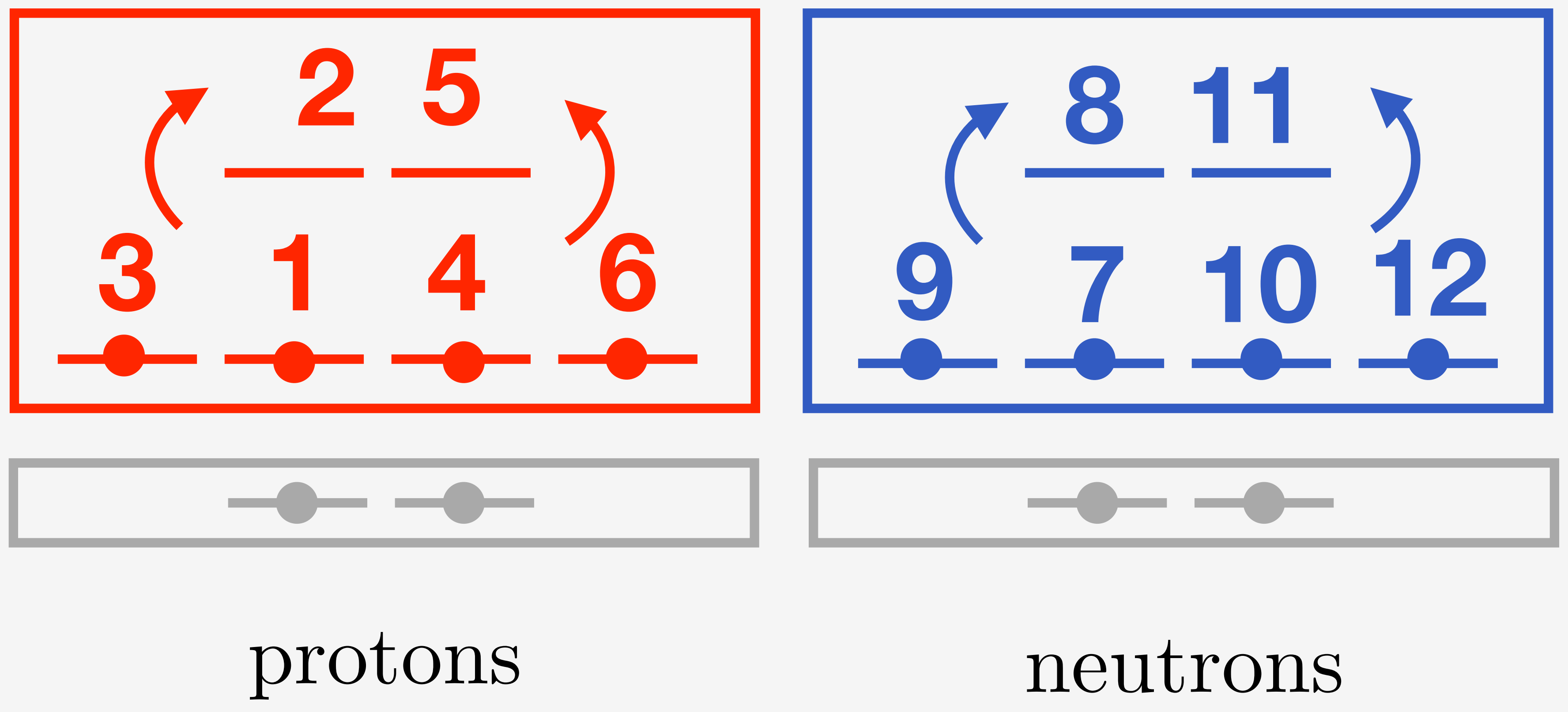
Magic via Stabilizer Rényi Entropy:

Leone, Oliviero, Hamma, PRL 128, 050402 (2022)

$$\mathcal{M}_\alpha(|\Psi\rangle) = -\log(d) + \frac{1}{1-\alpha} \log \left(\sum_P \frac{\langle \Psi | \hat{P} | \Psi \rangle^{2\alpha}}{d^\alpha} \right)$$

Jordan Wigner Mapping

$$a_i^\dagger \to \left(\prod_{j<i} \hat{\sigma}_z^{(j)}\right)\left(\hat{\sigma}_x^{(i)} - i\hat{\sigma}_y^{(i)}\right)/2$$



2 < Z,N < 8

12 qubits

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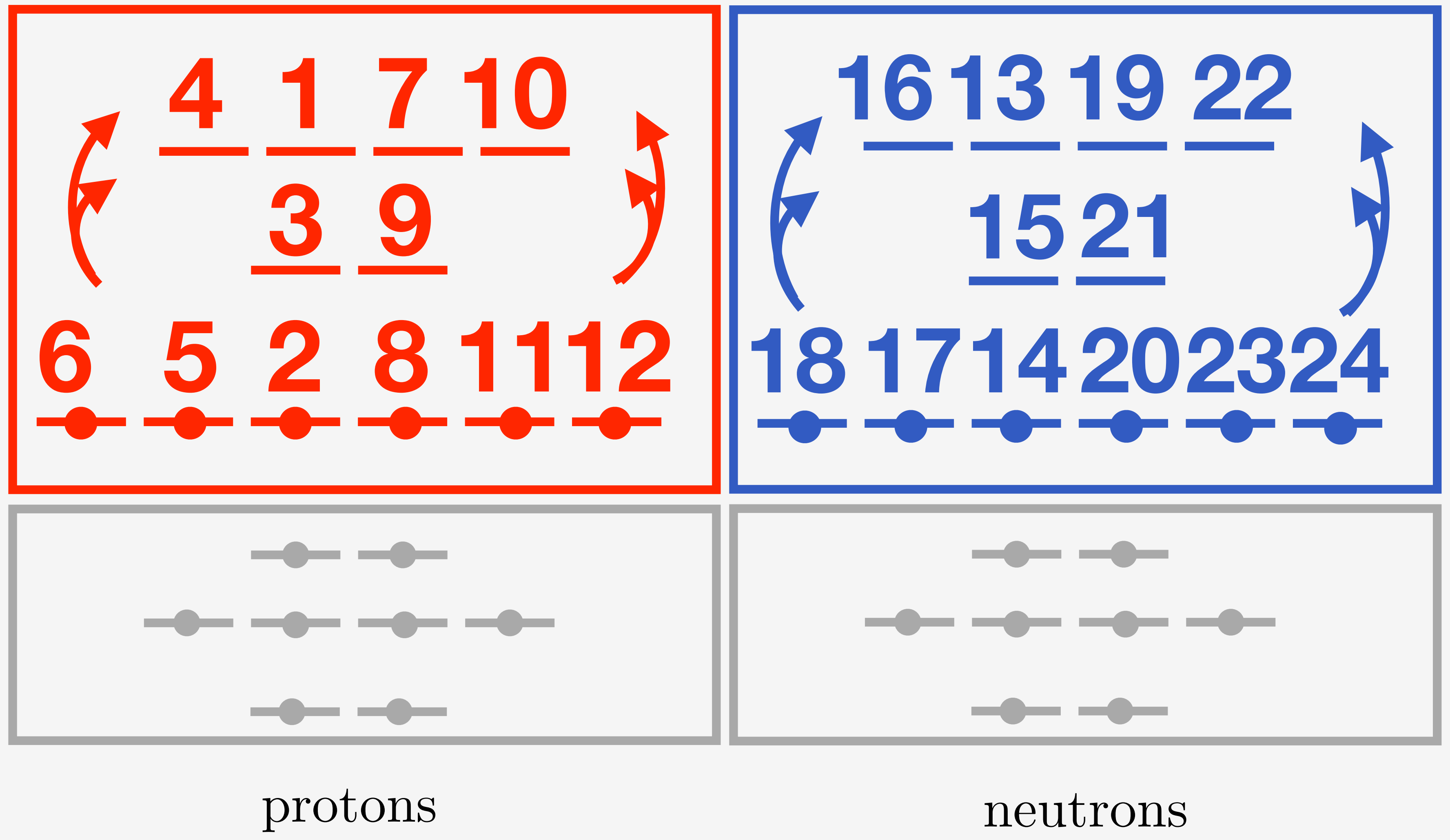
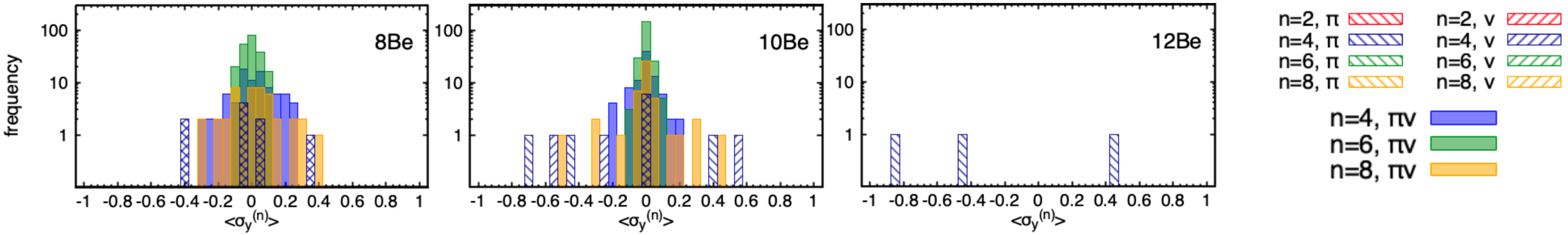


8 < Z,N < 20

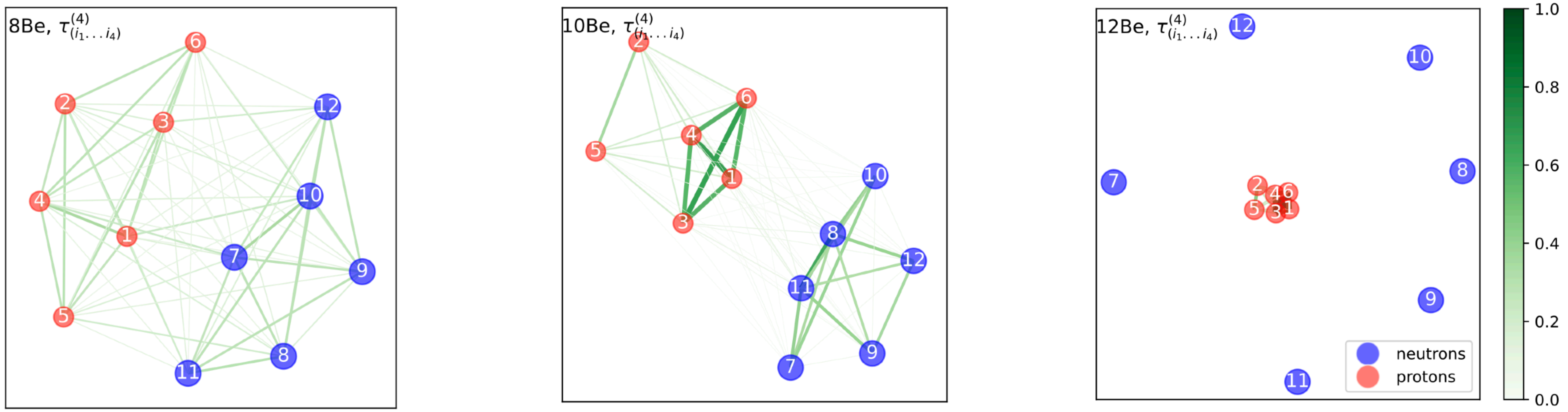
24 qubits

Multi-Partite Entanglement in 12-qubit Nuclei

✳*Distribution of the IY Pauli strings expectation values*



✳*Network plots*



edge values

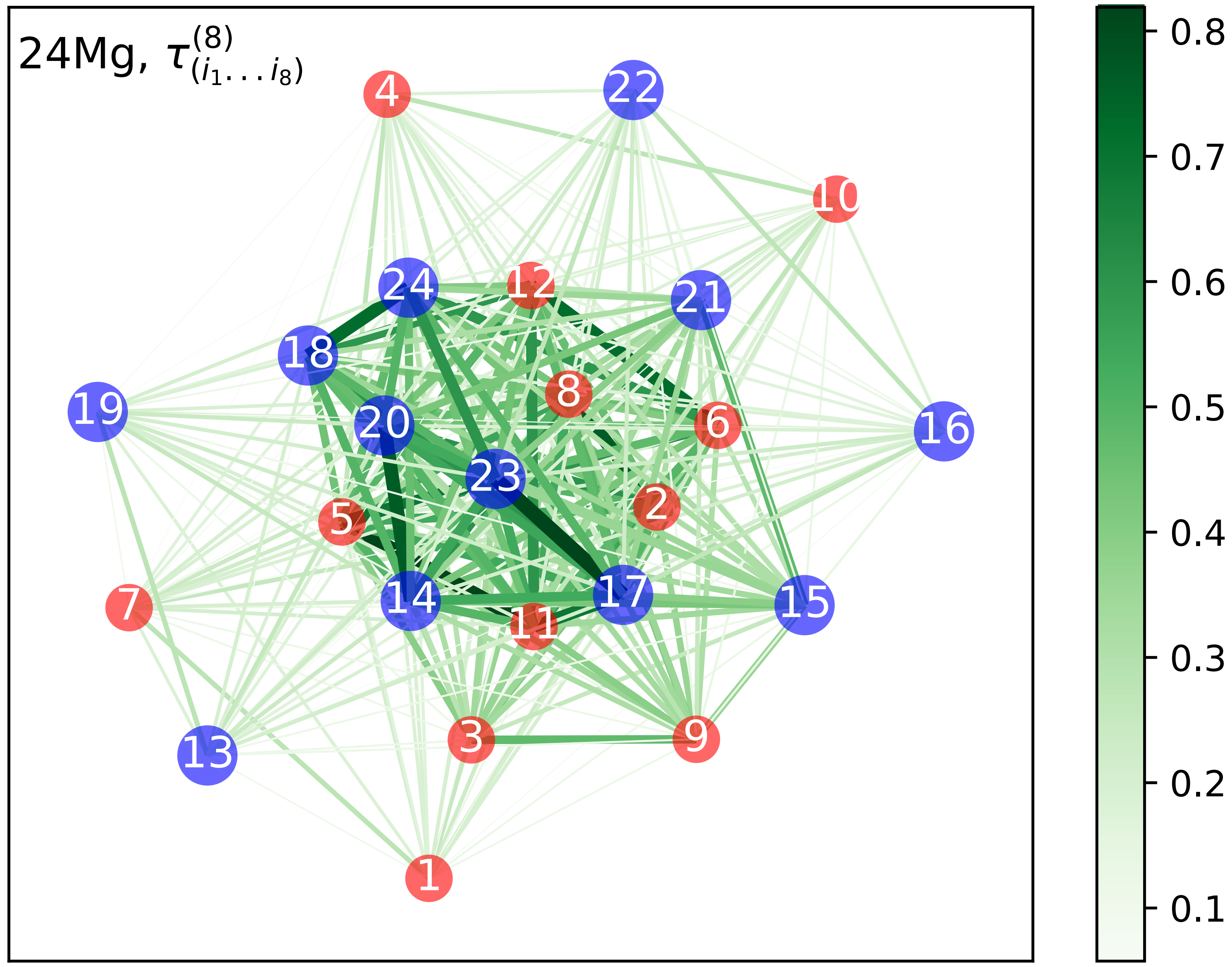
$$e^{(4)}_{i_1 i_2} = \sum_{i_3 < i_4} \tau^{(4)}_{(i_1,i_2,i_3,i_4)}$$

- large many-body entanglement when the model space and symmetries allow it
- proton-neutron entanglement is more collective than pure proton or neutron entanglement
- protons become more entangled with neutron excess

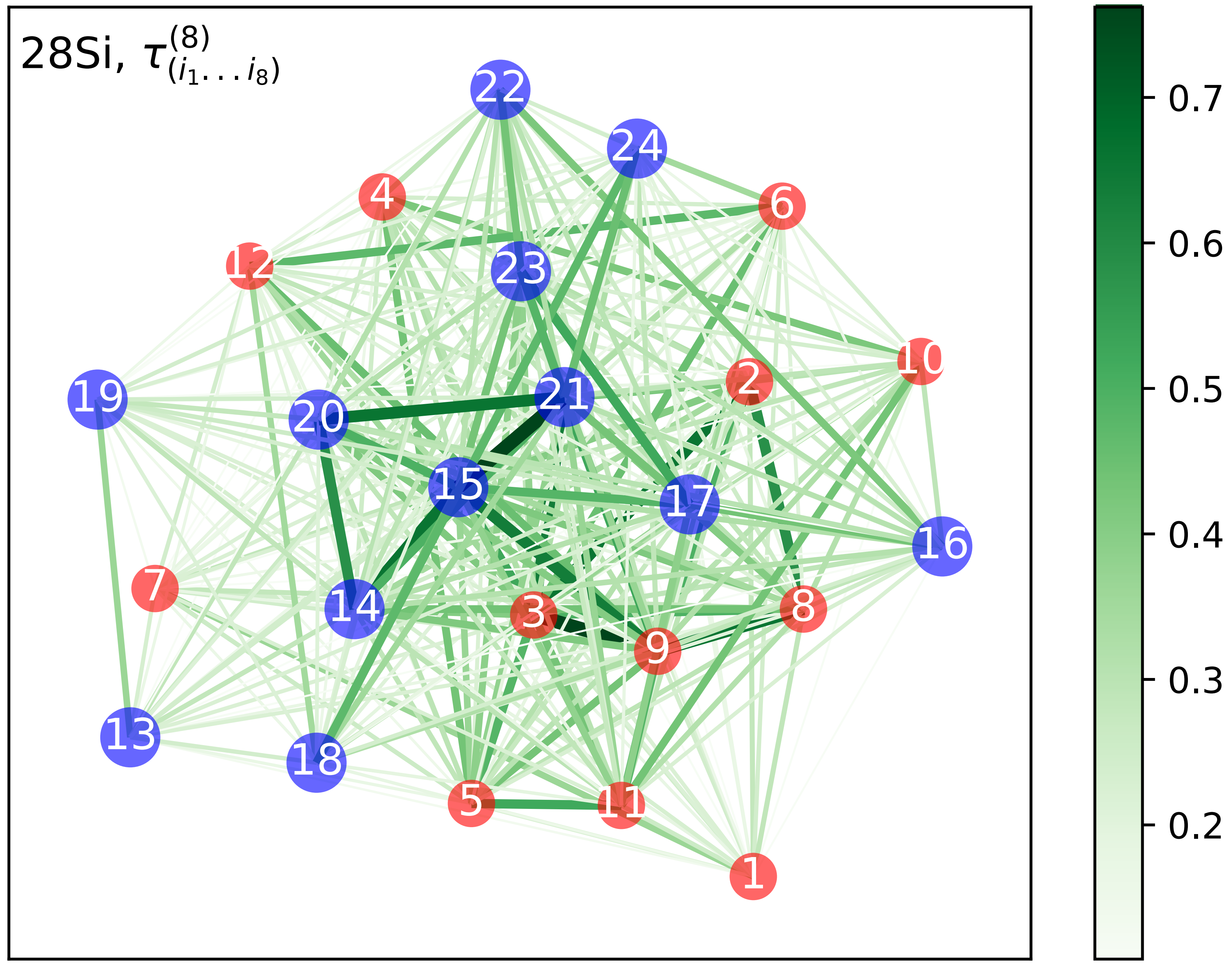
arXiv:2409.12064

Multi-Partite Entanglement in 24-qubit Nuclei

$$e^{(8)}_{i_1 i_2} = \sum_{i_3<i_4<i_5<i_6<i_7<i_8} \tau^{(8)}_{(i_1,i_2,i_3,i_4,i_5,i_6,i_7,i_8)}$$



N=Z=12



N=Z=14

large proton-neutron 8-tangles → hint of alpha correlations?

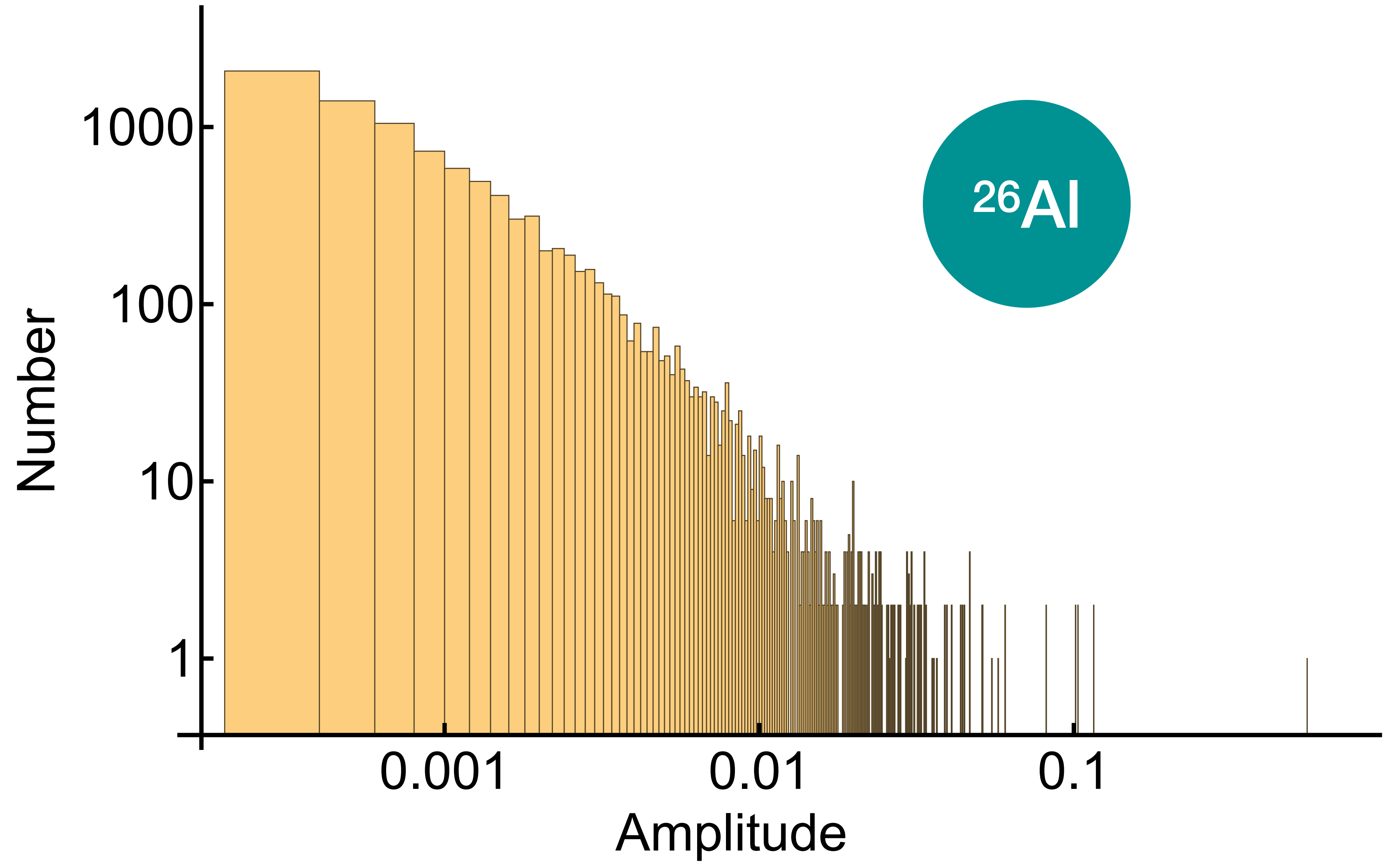
arXiv:2409.12064

Magic in Nuclei: PSIZe-MCMC algorithm

- SREs require $d^2 = 4^{n_{qubits}}$ expectation values

- MCMC techniques can be used to compute SREs in large systems *Tarabunga et al PRX Quantum 4, 040317 (2023)*

- However the distribution of amplitudes in the wave function of collective nuclei slows down the convergence of MCMC

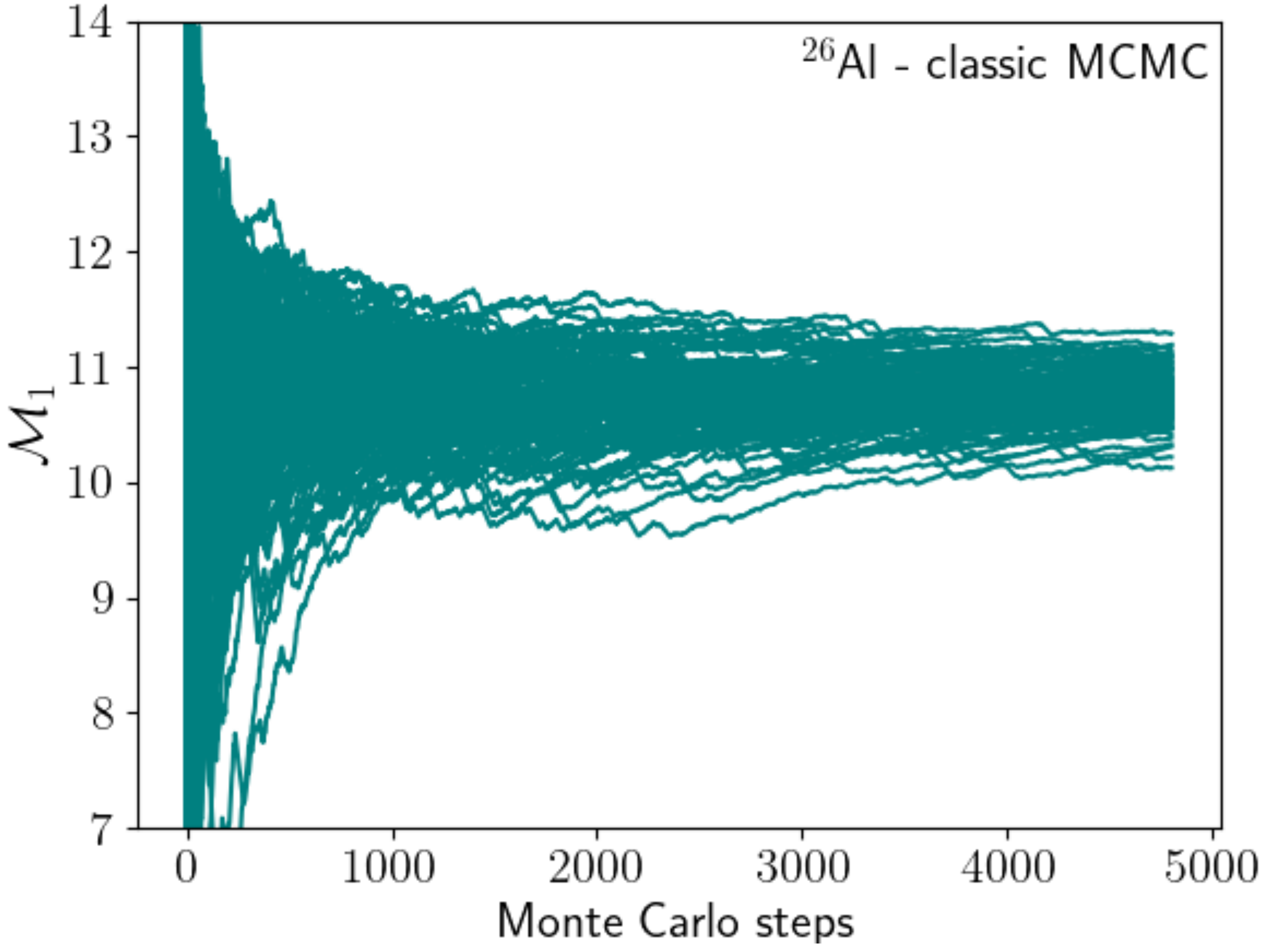


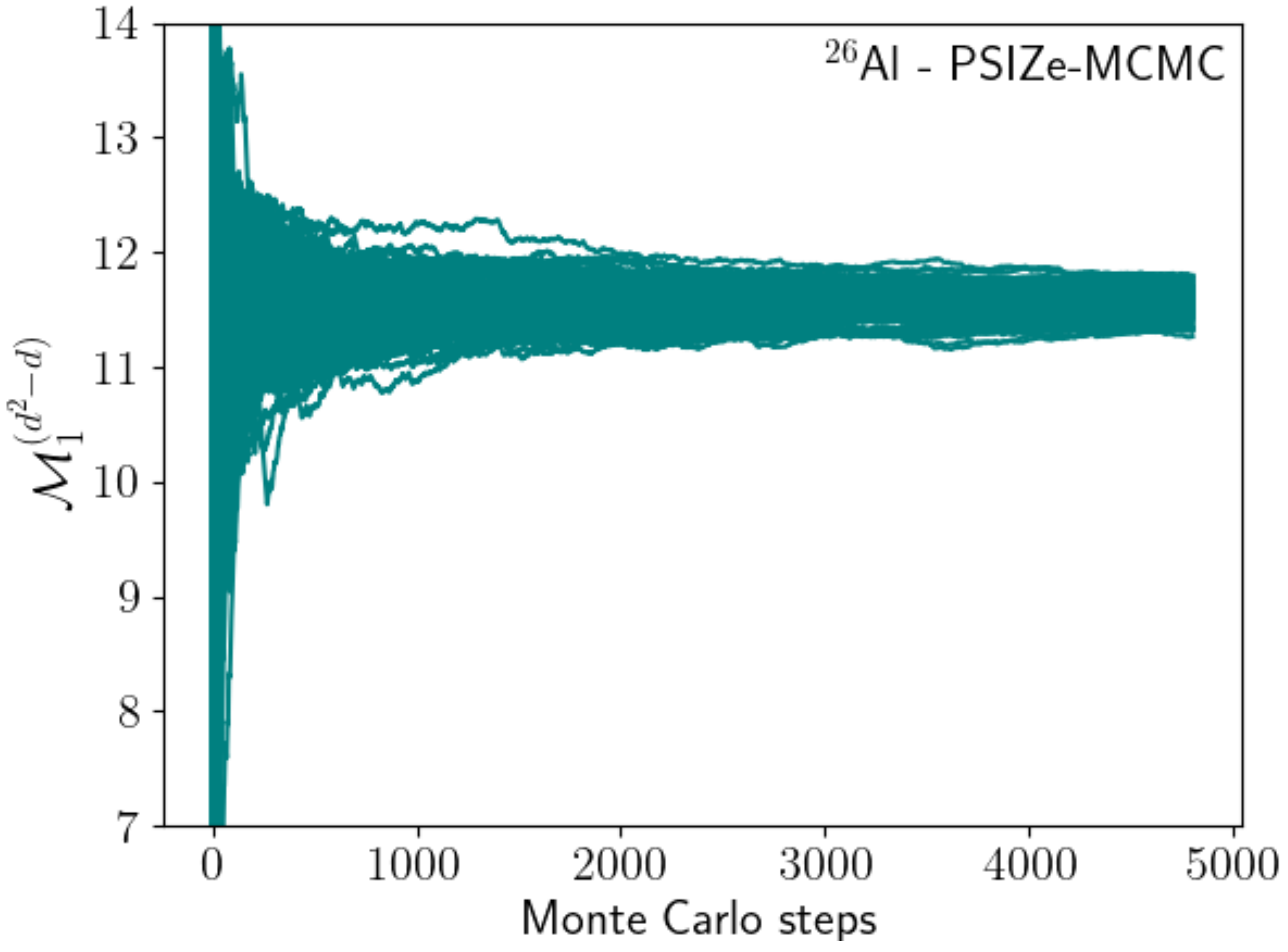
➡ "Pauli-String IZ exact MCMC" (PSIZe-MCMC) algorithm:

Expectation values of IZ strings computed exactly, MCMC to sample the remaining space

arXiv:2409.12064

Magic in Nuclei: PSIZe-MCMC algorithm





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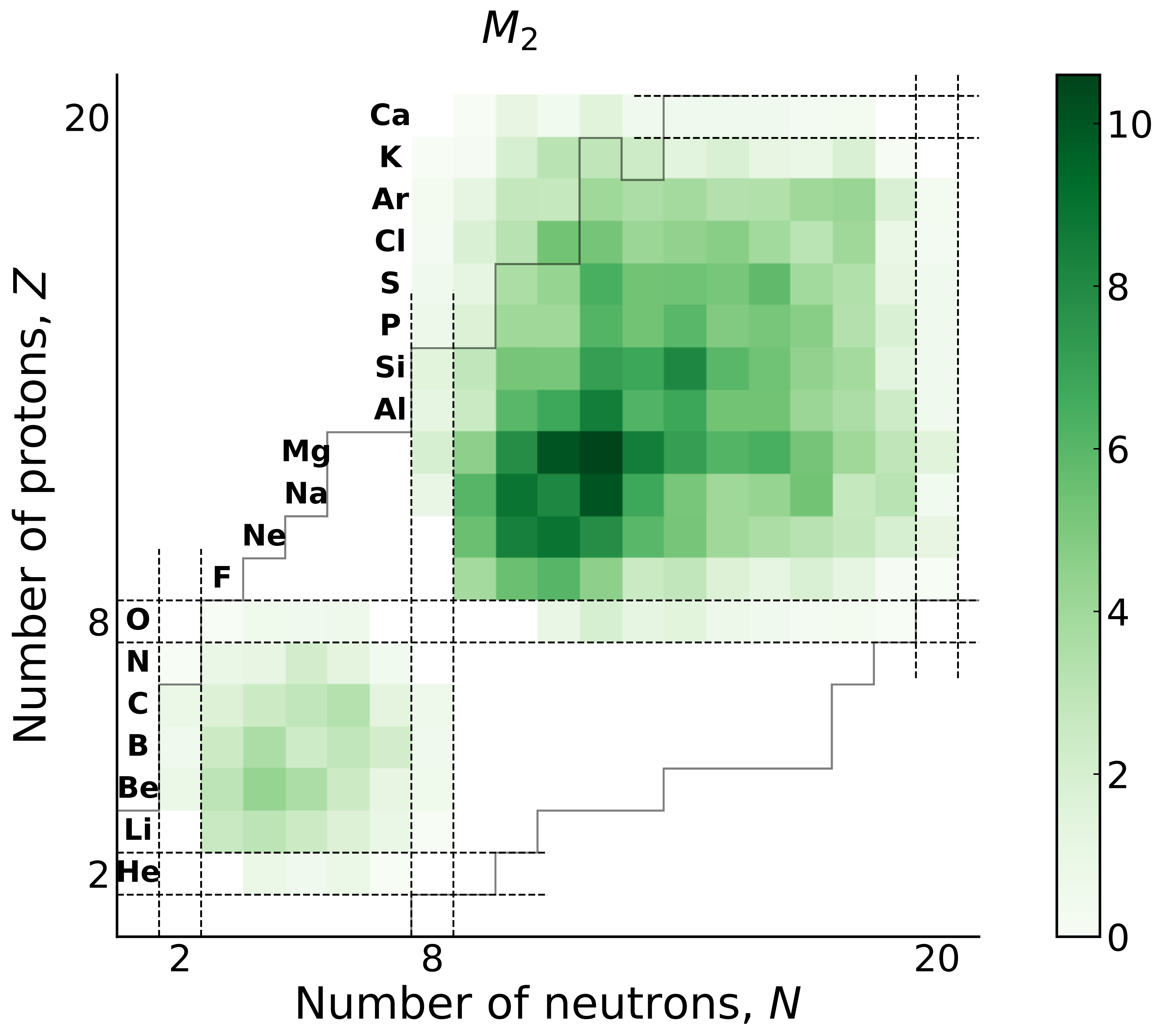
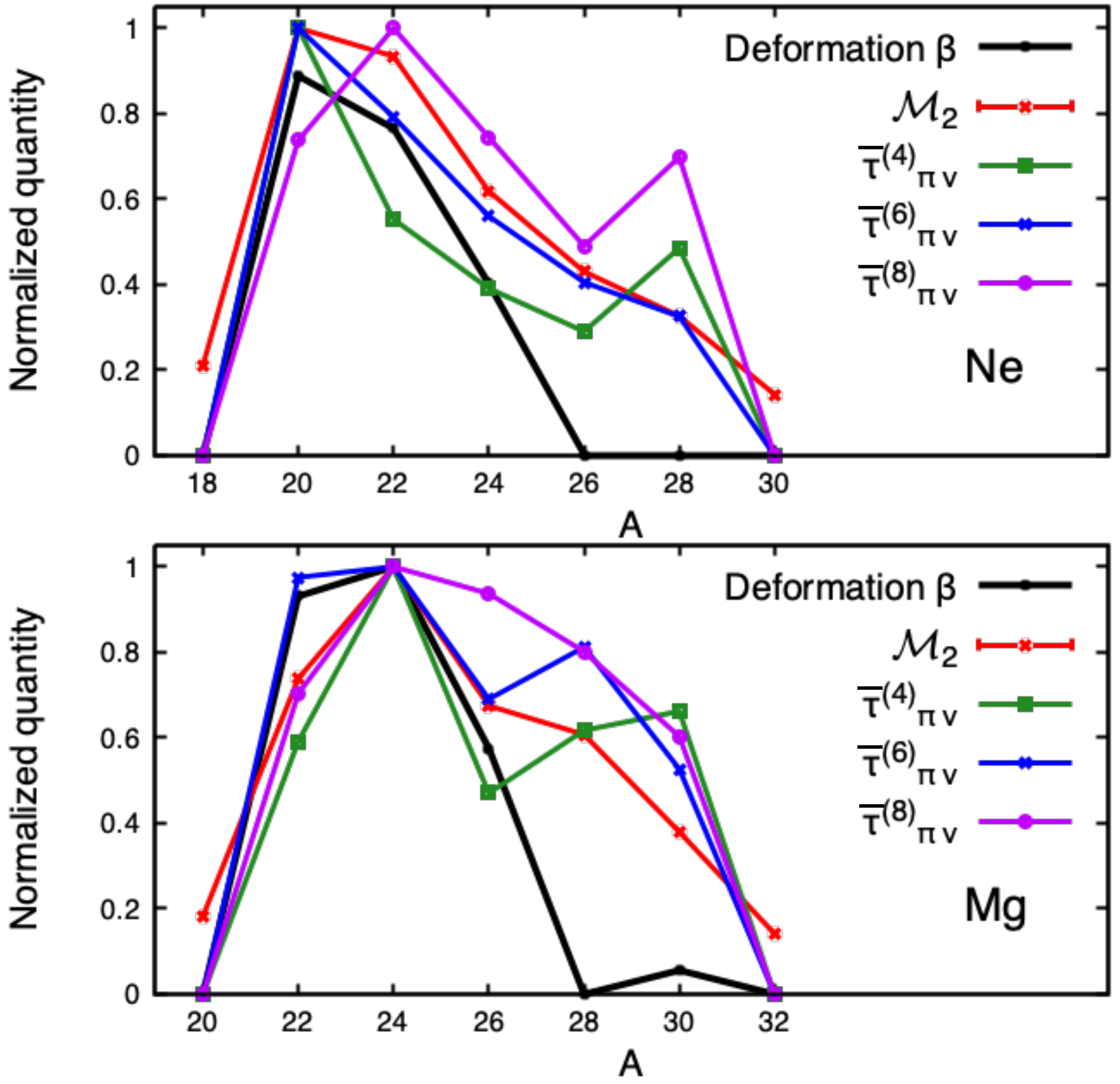
Magic and Entanglement in Nuclei

$$\overline{\tau}^{(n)}_{\pi,\nu,\pi\nu} \equiv \sum_{i_1,i_2,\dots i_n \in \pi,\nu,\pi\nu} \tau^{(n)}_{(i_1 \dots i_n)}$$

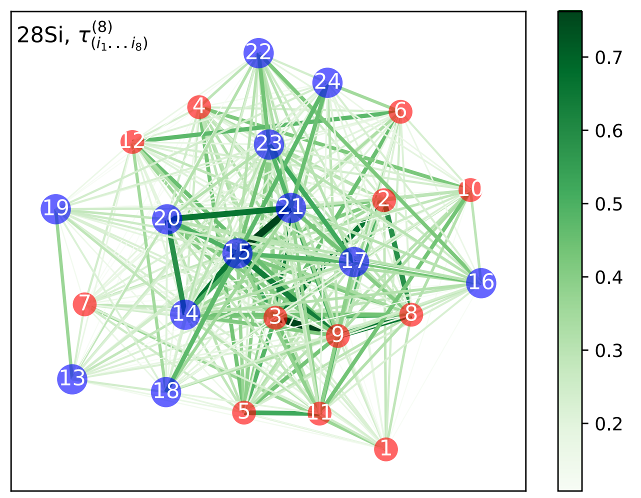




- Maximal magic and proton-neutron tangles coincides with maximal deformation in nuclei
- Magic and tangles also persist in the region where axial deformation vanishes (shape co-existence)

arXiv:2409.12064

Outline



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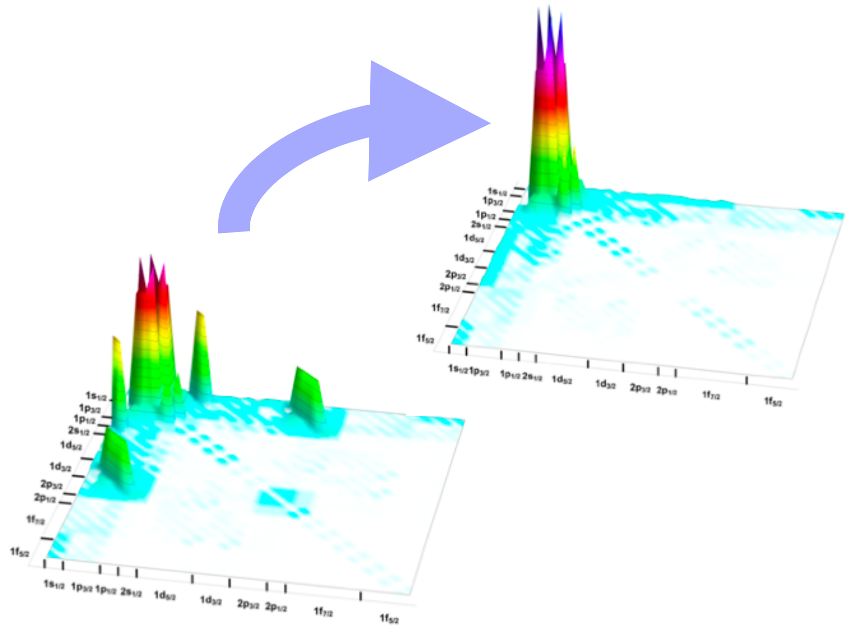

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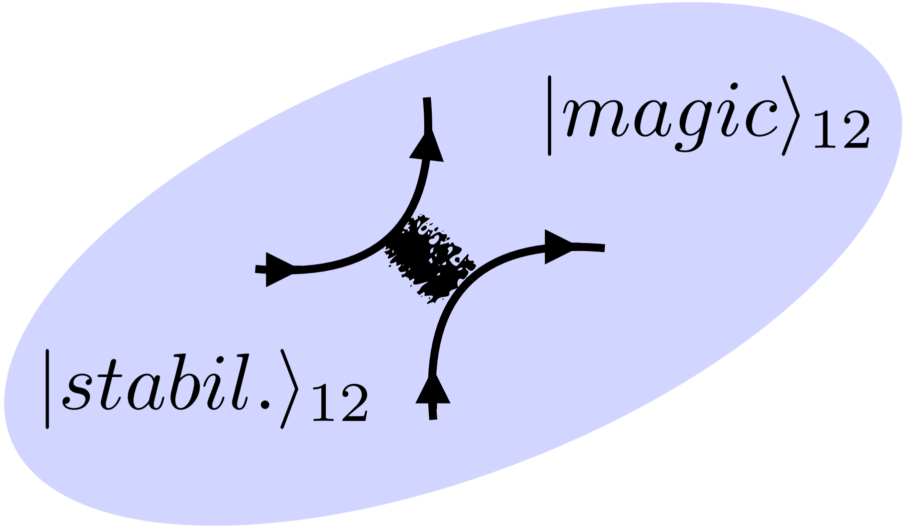

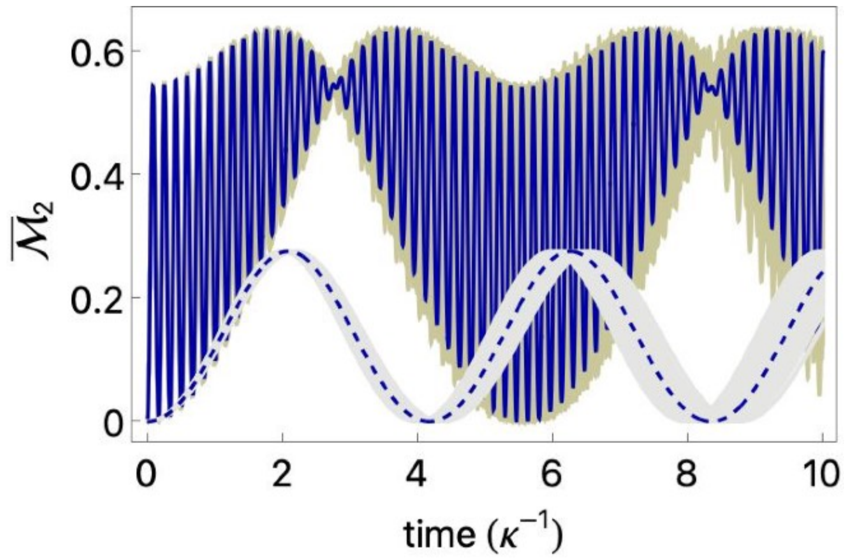

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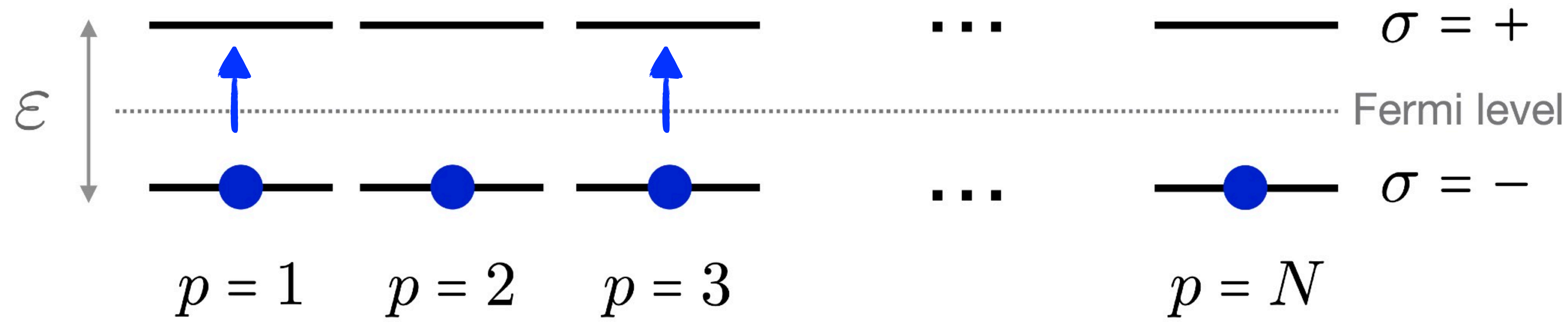
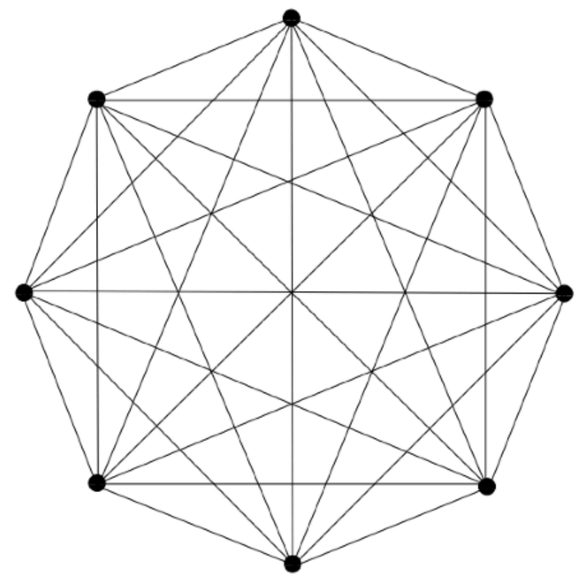
Chernyshev, CR, Savage arXiv:2411.04203

The Lipkin-Meshkov-Glick Model: a sandbox for new ideas

Lipkin, Meshkov, Glick, Nucl. Phys. 62, 188 (1965)





all-to-all connectivity

$$H = \varepsilon J_z - \frac{V}{2}(J_+^2 + J_-^2)$$

$$J_z = \frac{1}{2}\sum_{p\sigma} \sigma c^\dagger_{p\sigma} c_{p\sigma}$$

$$J_+ = \sum_p \sigma c^\dagger_{p+} c_{p-}\,, \qquad J_- = (J_+)^\dagger$$

Relevance for many-body physics, trapped-ion quantum computing, spin squeezing…

▶ Benchmark for studying relations between entanglement and quantum phase transitions

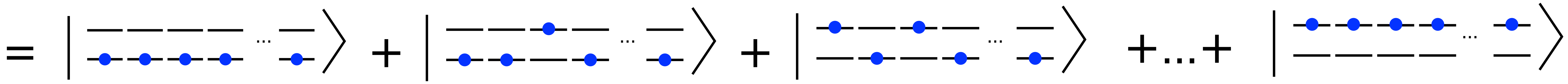
See e.g. J. Vidal et al. PRA 69, 022107 & 054101 (2004); Di Tullio et al, PRA 100, 062104 (2019); Faba, Martín, Robledo, PRA, 103, 032426 (2021); PRA 104, 032428 (2021); PRA 105, 062449 (2022); Hengstenberg, CR, Savage EPJA 59, 231 (2023)…

▶ for testing and comparing new quantum algorithms:

Cervia et al. PRC 104, 024305 (2021); Chikaoka & Liang, Chin. Phys. C 46 024106 (2022); Romero et al. PRC 105, 064317 (2022); Hlatshwayo et al. PRC 106, 024319 (2022); Robin, Savage PRC 108, 024313 (2023); Beaujeault-Taudiere, Lacroix, arXiv:2312.04703 (2023); Hlatshwayo et al. PRC 109, 014306 (2024)…

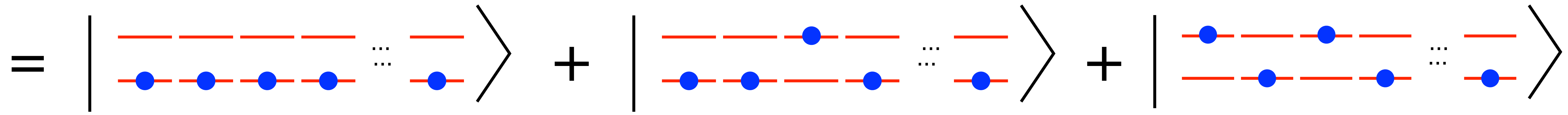
The Lipkin-Meshkov-Glick Model in Effective Model Spaces

✳ ***Exact solution:*** $|\Psi\rangle = \sum_{M=-J}^{J} A_{J,M}|J,M\rangle \equiv \sum_{n=0}^{2J} A_n|n\rangle$

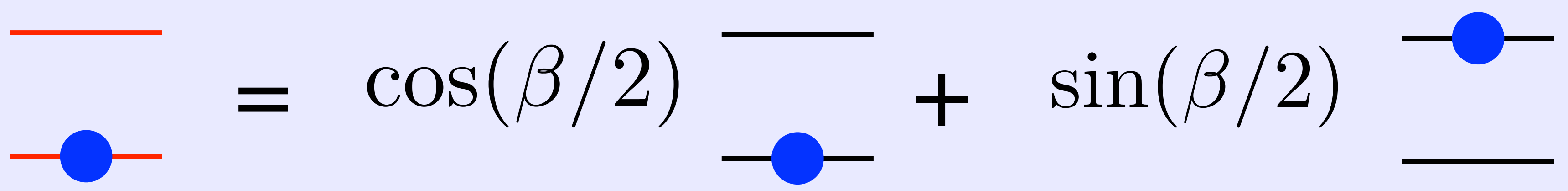


✳ ***Effective description:***

$|\Psi\rangle^{\Lambda} = \sum_{n=0}^{\Lambda-1} A_n|n\rangle$



Rotation of the spins as "disentanglers"



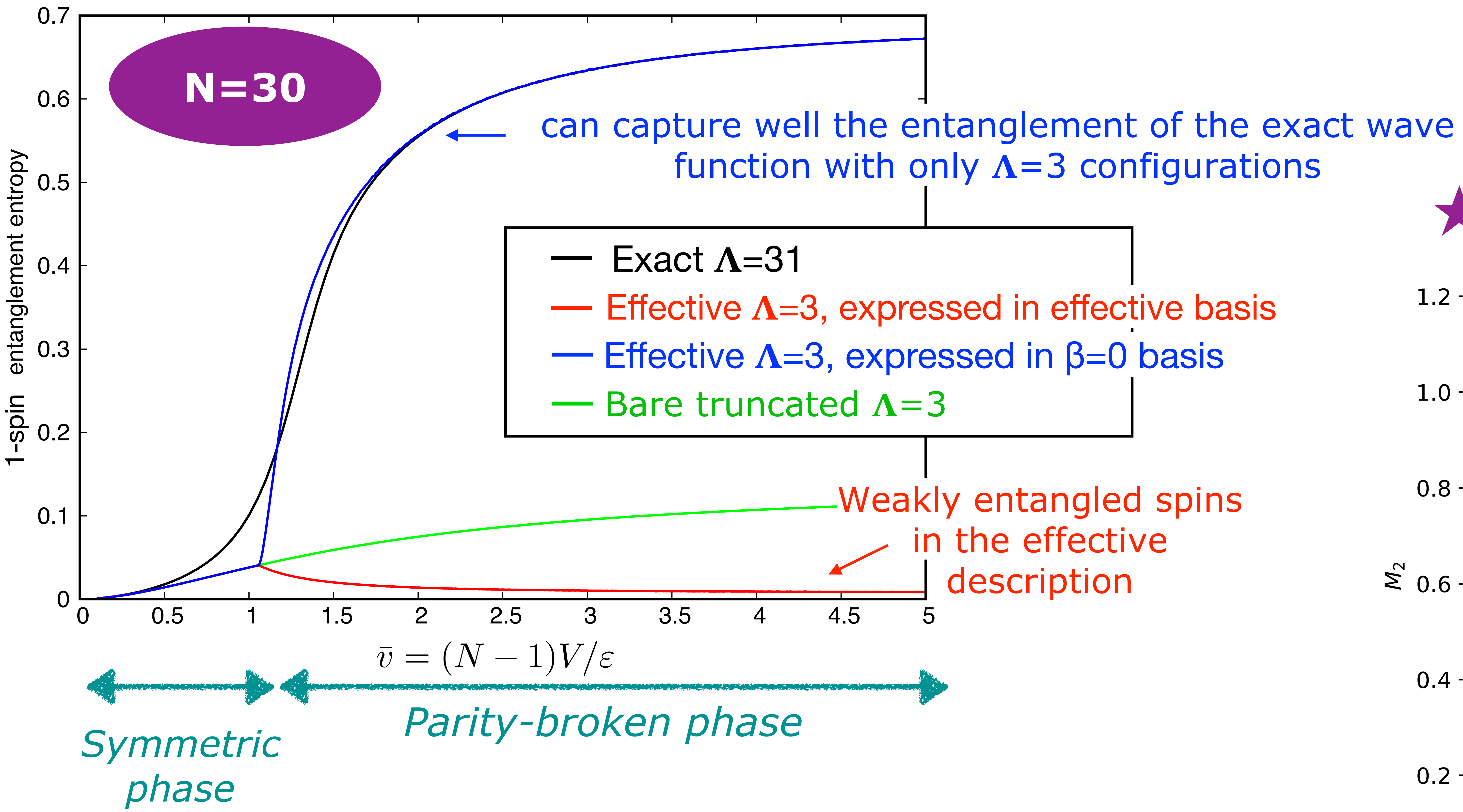
Effective Hamiltonian $H(\beta) = U(\beta)^{\dagger} H U(\beta)$ *with* $U(\beta) = e^{-iJ_y\beta}$

Determined from energy minimization

Similar but different technique used in tensor networks to disentangle the sites:
MERA Vidal 2007, in combination with MPS -> yesterday's talks by Mingpu Qin and Gerald Fux

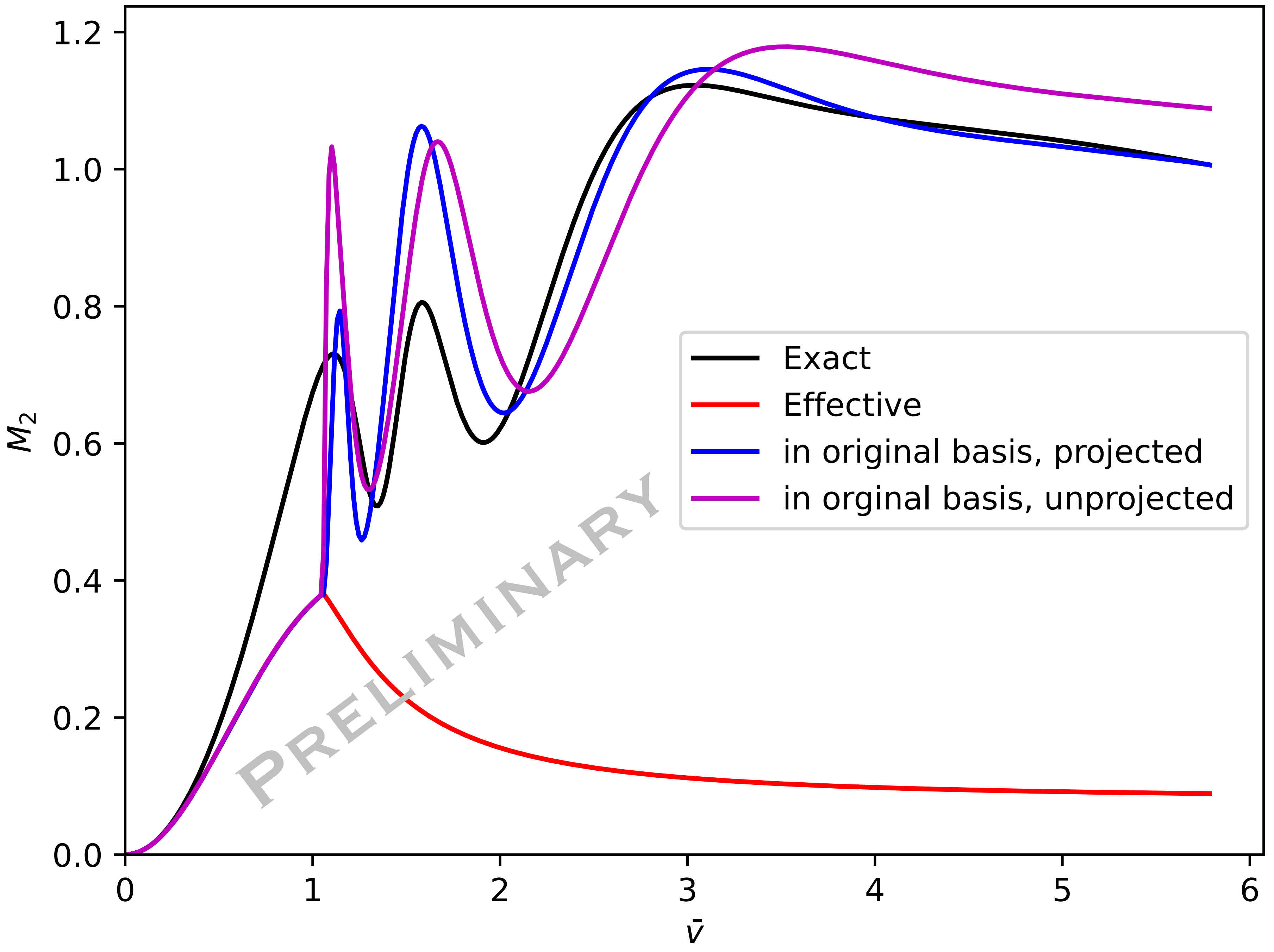
Entanglement and Magic Rearrangement

★ 1-spin entanglement entropy



Hengstenberg, CR, Savage EPJA 59, 231 (2023)

★ Stabilizer Rényi entropy

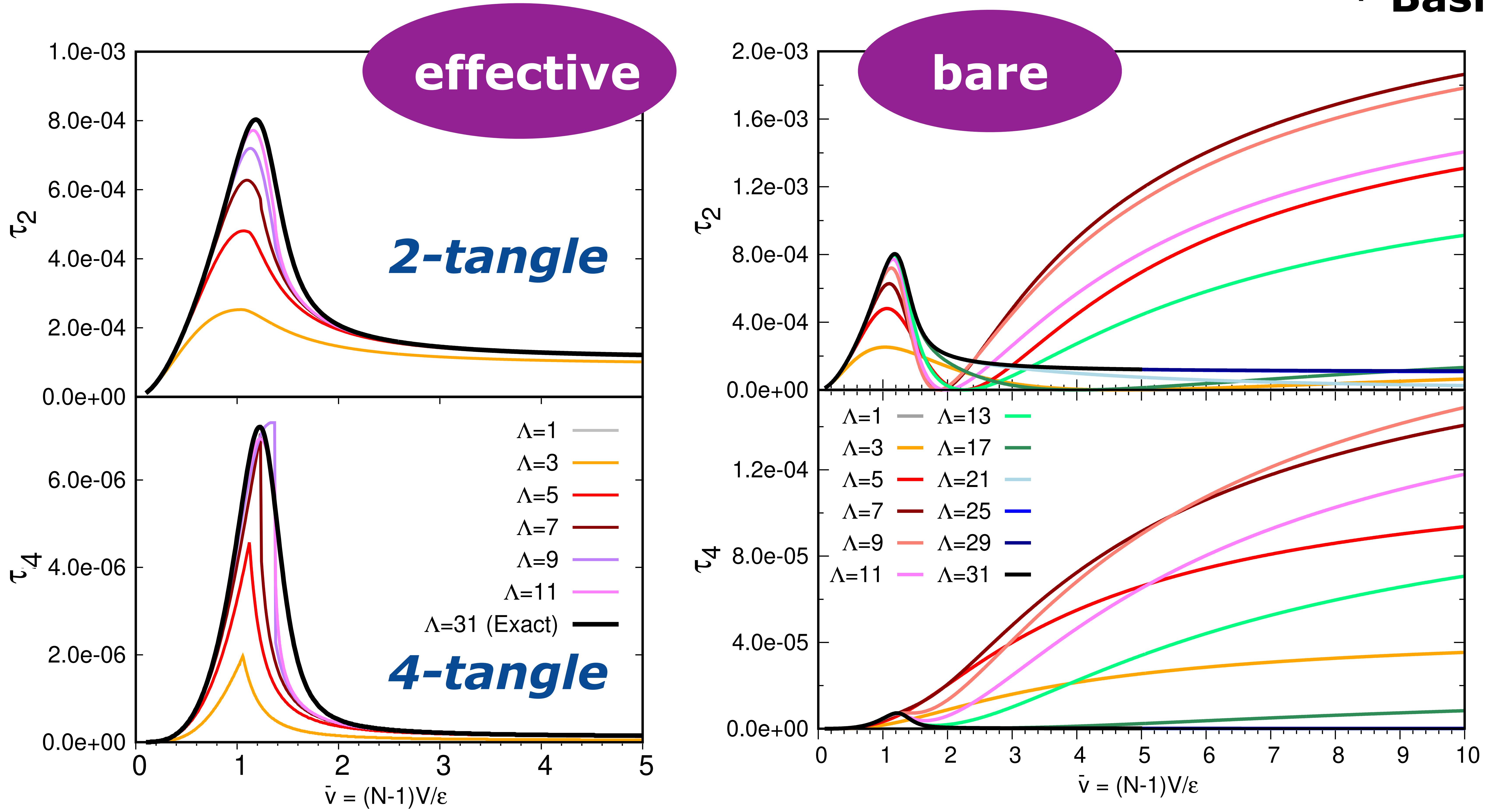


Sensitivity of multi-body entanglement to truncation and optimization

★ **n-tangles** $\tau_n = \left| \langle \Psi | \hat{\sigma}_y^{\otimes n} | \Psi^* \rangle \right|^2$

Multi-"spin" entanglement

*** Basis independent ***



Effective: *Rapid convergence which can be further improved with projection*

Bare: *convergence badly behaved*

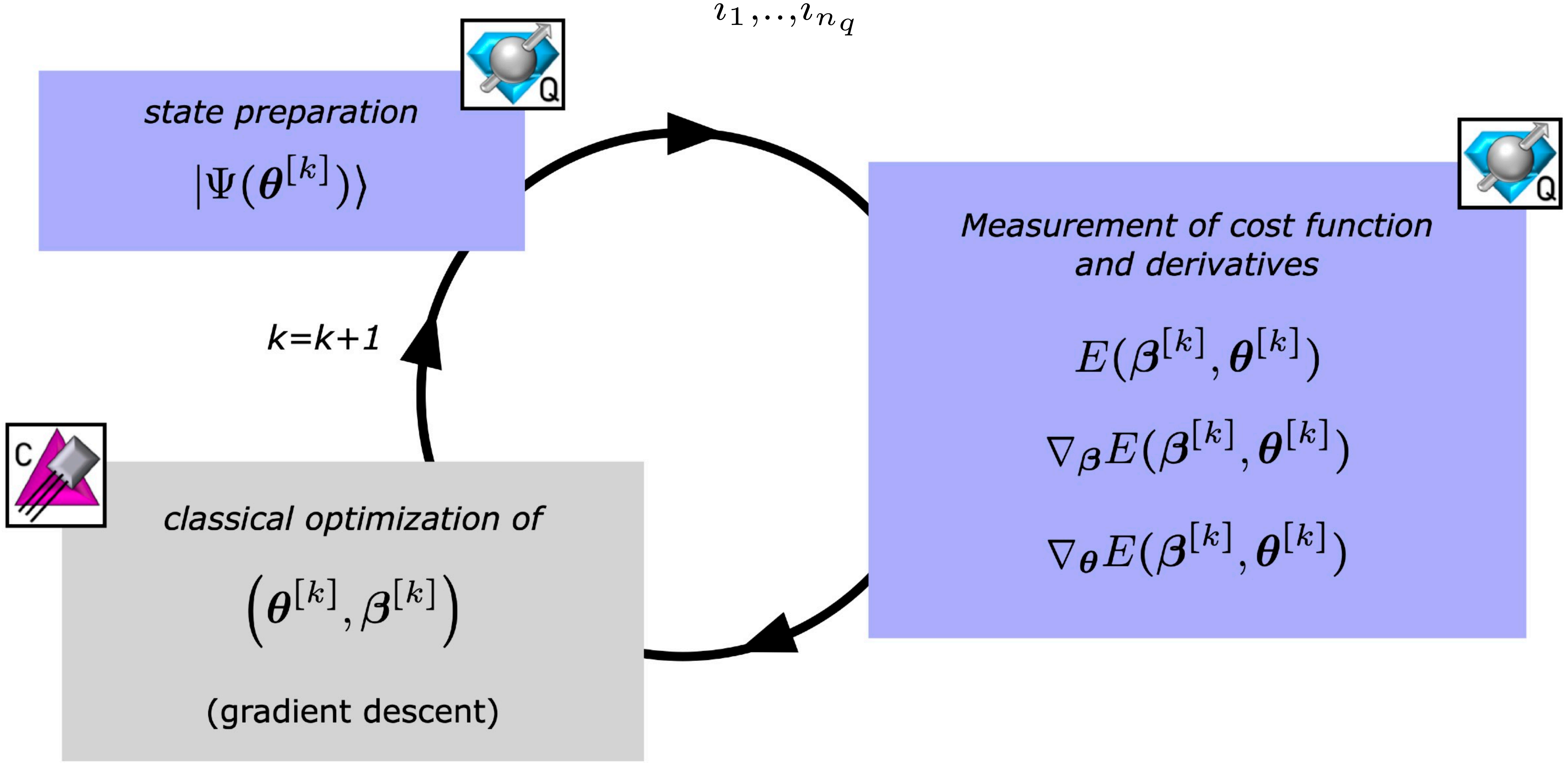
EPJA 59, 231 (2023)

★ *Hamiltonian-Learning-VQE Algorithm:*

CR, Savage PRC 108, 024313 (2023)

Cost function to minimize: $E(\beta, \boldsymbol{\theta}) = \langle \Psi(\boldsymbol{\theta}) | \hat{H}(\beta) | \Psi(\boldsymbol{\theta}) \rangle$

$$= \sum_{i_1,..,i_{n_q}} h_{i_i,..,i_{n_q}}(\beta) \, \langle \Psi(\boldsymbol{\theta}) | \overline{\sigma}_{i_1} \otimes \ \ldots \otimes \overline{\sigma}_{i_{n_q}} | \Psi(\boldsymbol{\theta}) \rangle$$



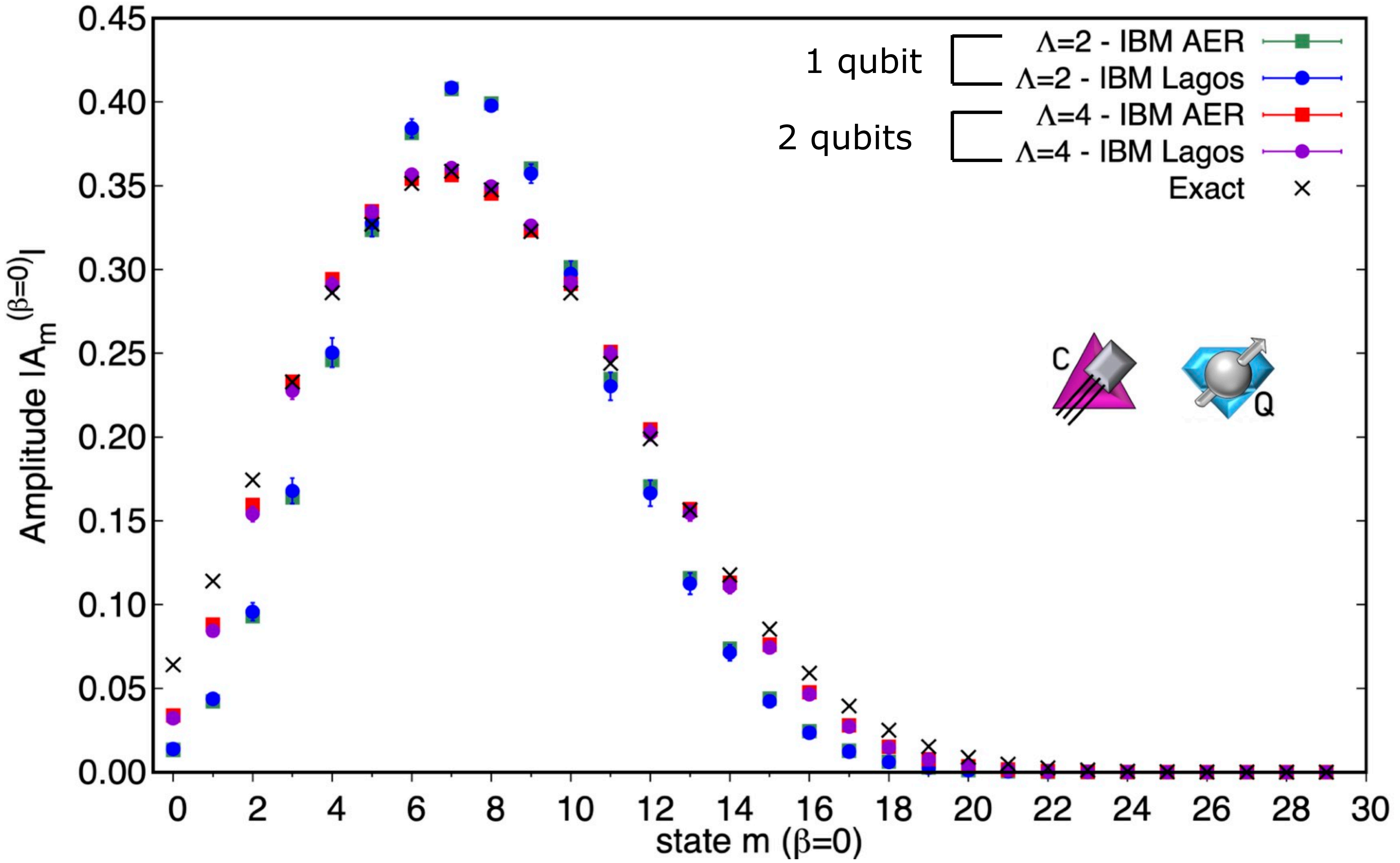
⇒ learns the effective Hamiltonian and identifies the associated ground state simultaneously

Entanglement and Magic Rearrangement in Quantum Simulations

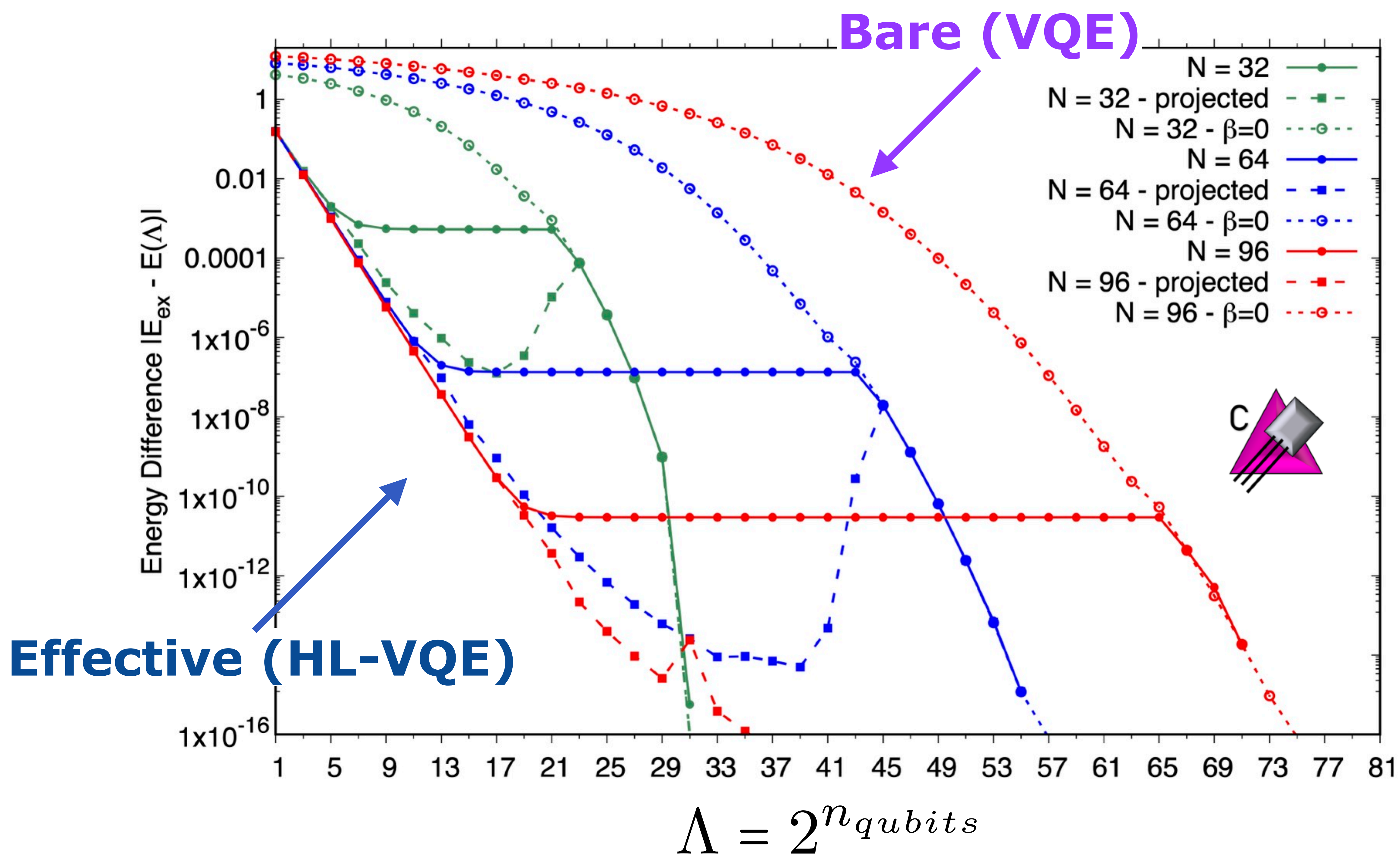
CR, Savage PRC 108, 024313 (2023)

★ Hamiltonian-Learning-VQE Algorithm:

Wave function extracted from IBM quantum computer



Exponential Acceleration in the expected convergence:



Exponentially-less resources are needed for given accuracy

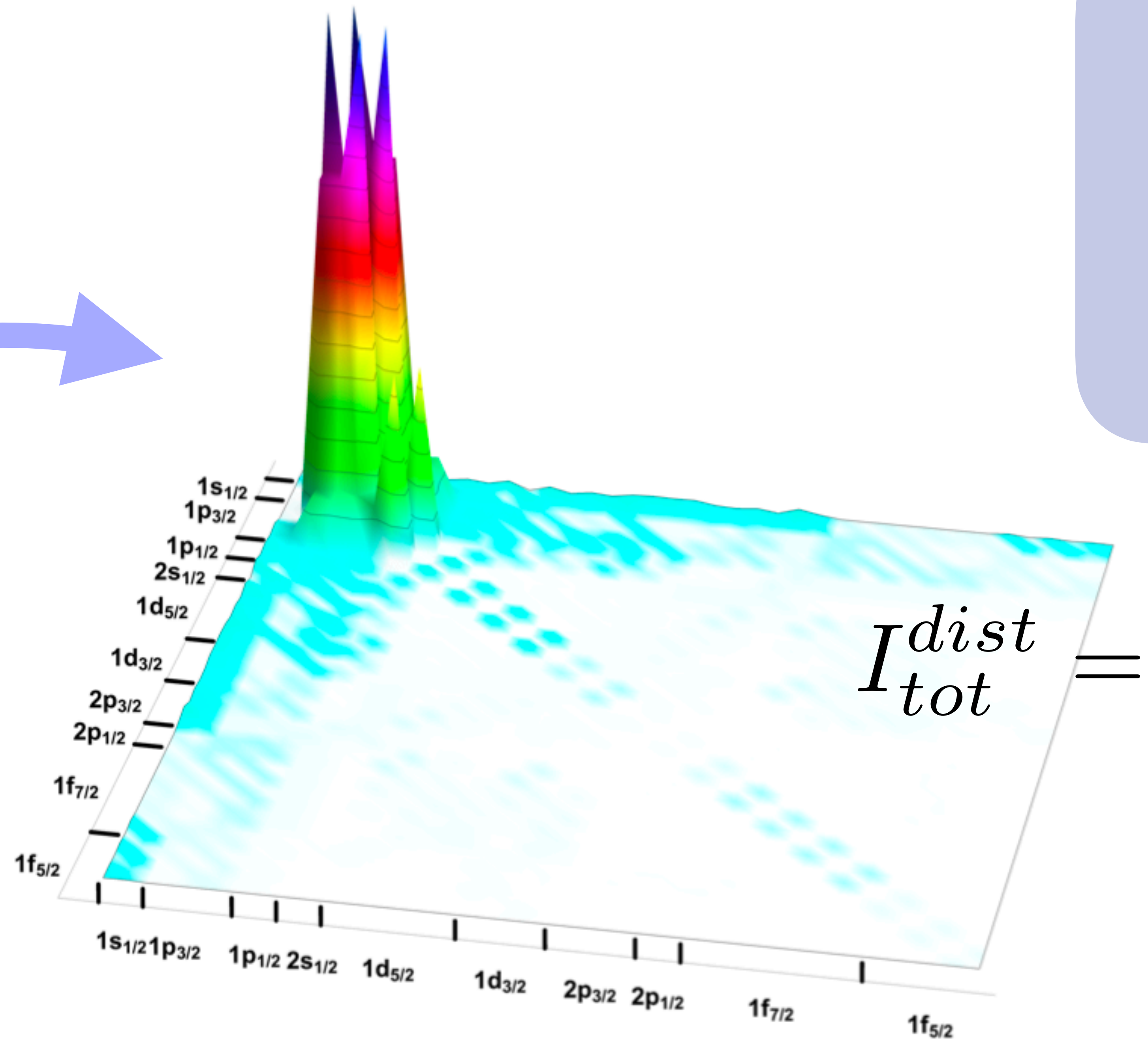
Entanglement Rearrangement In Nuclei

"Entanglement distance":

$$I_{ij}^{dist} = I_{ij} \times |i-j|^2$$

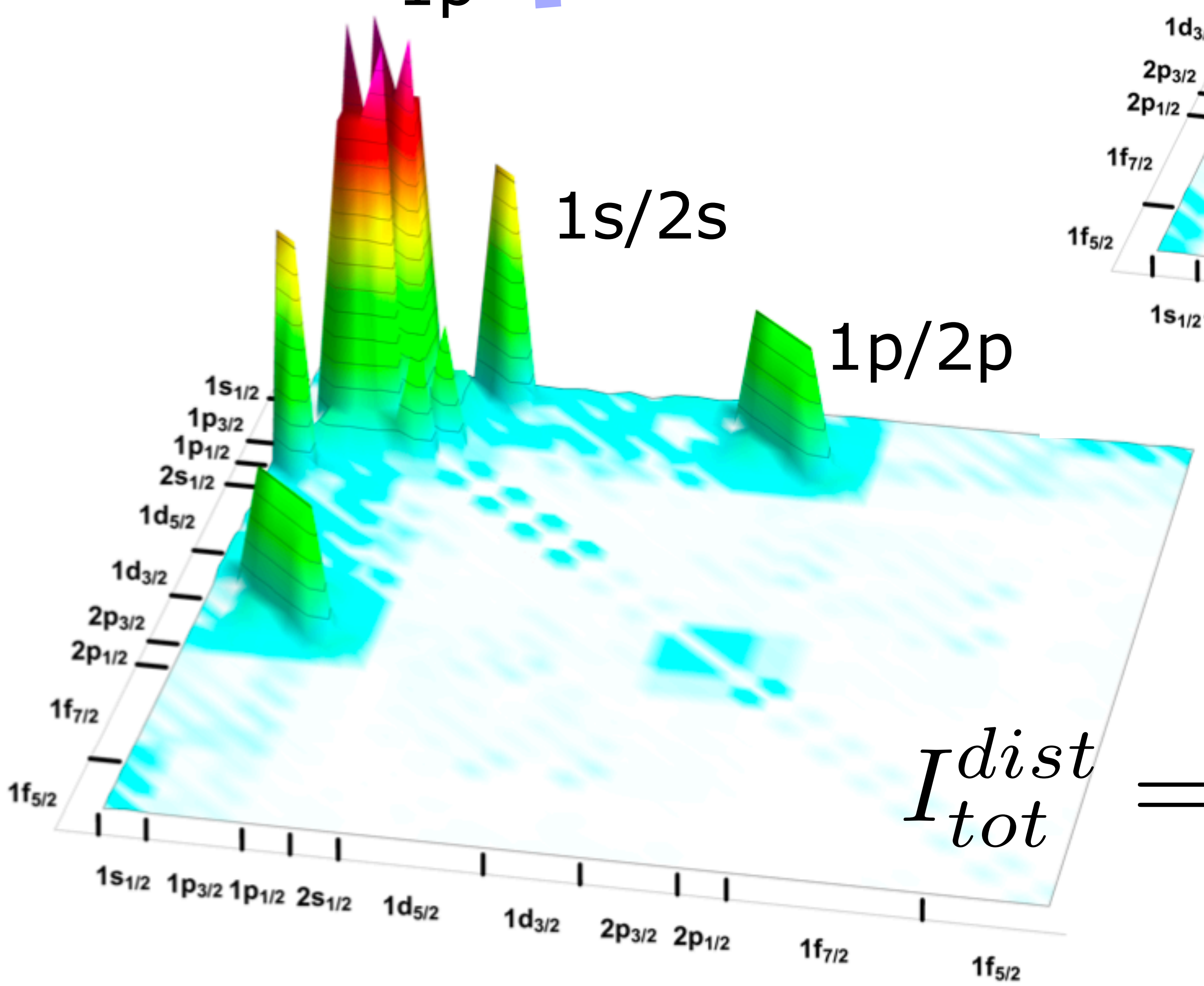
$$I_{tot}^{dist} = \sum_{ij} I_{ij}^{dist}$$

Orbital optimization



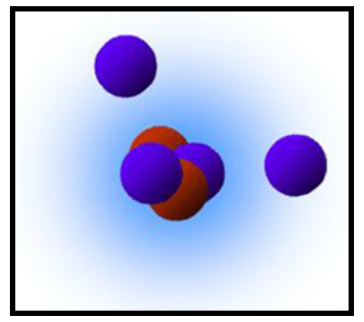
$$I_{tot}^{dist} = 115.90$$

⁶He



Neutron-neutron mutual information

$$I_{tot}^{dist} = 565.24$$



⇒ emergence of ^{4}He-core + nn-valence structure?

CR, Savage, Pillet, PRC 103, 034325 (2021)

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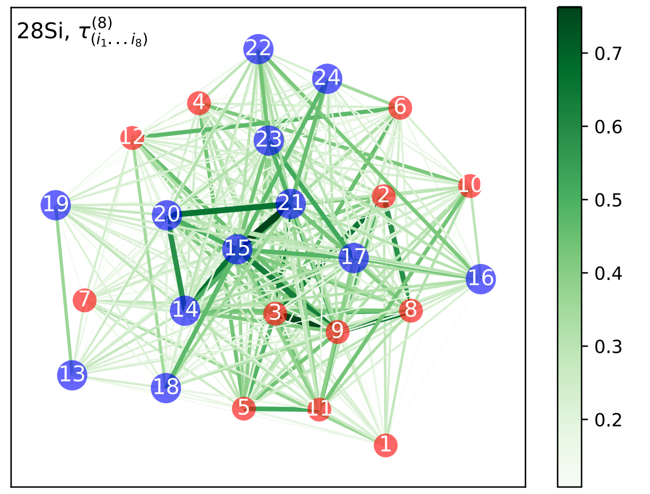
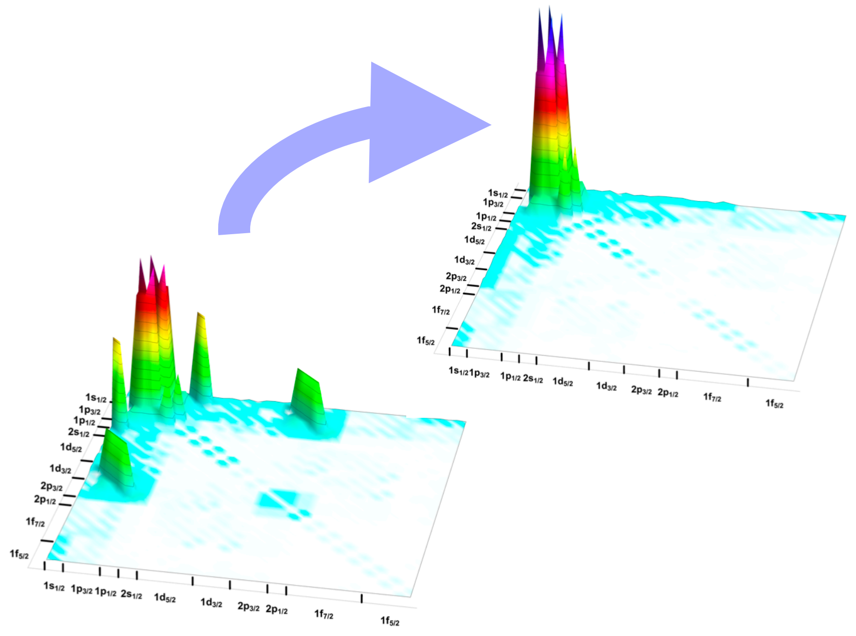
Brökemeier, Hengstenberg, Keeble, CR, Rocco & Savage, arXiv:2409.12064





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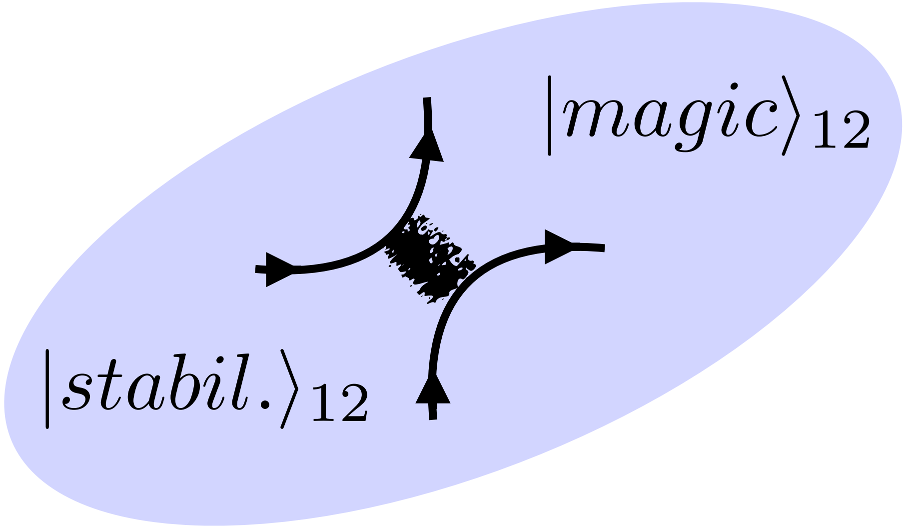

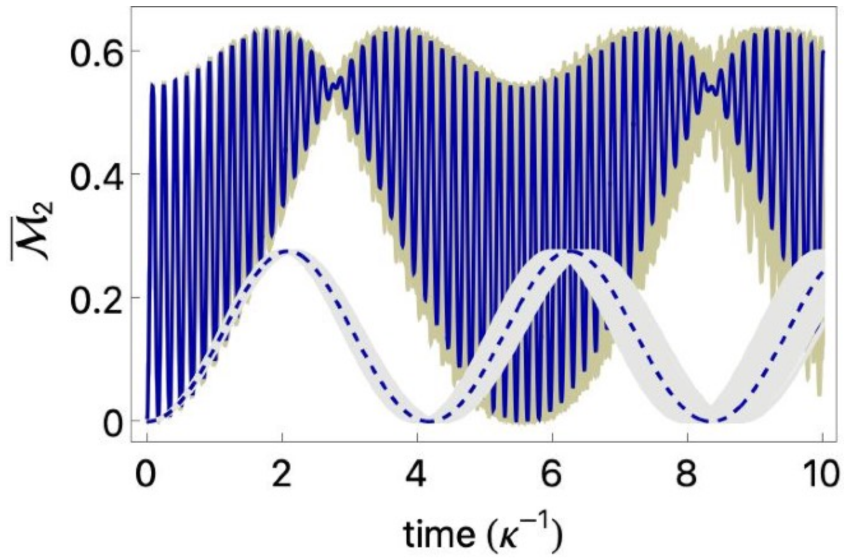

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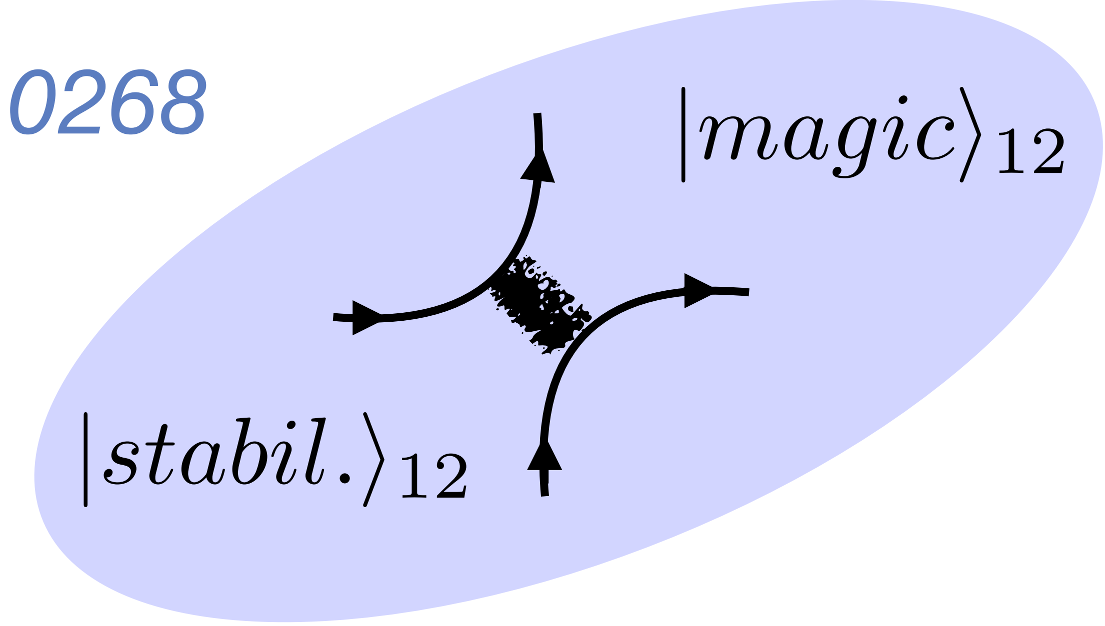
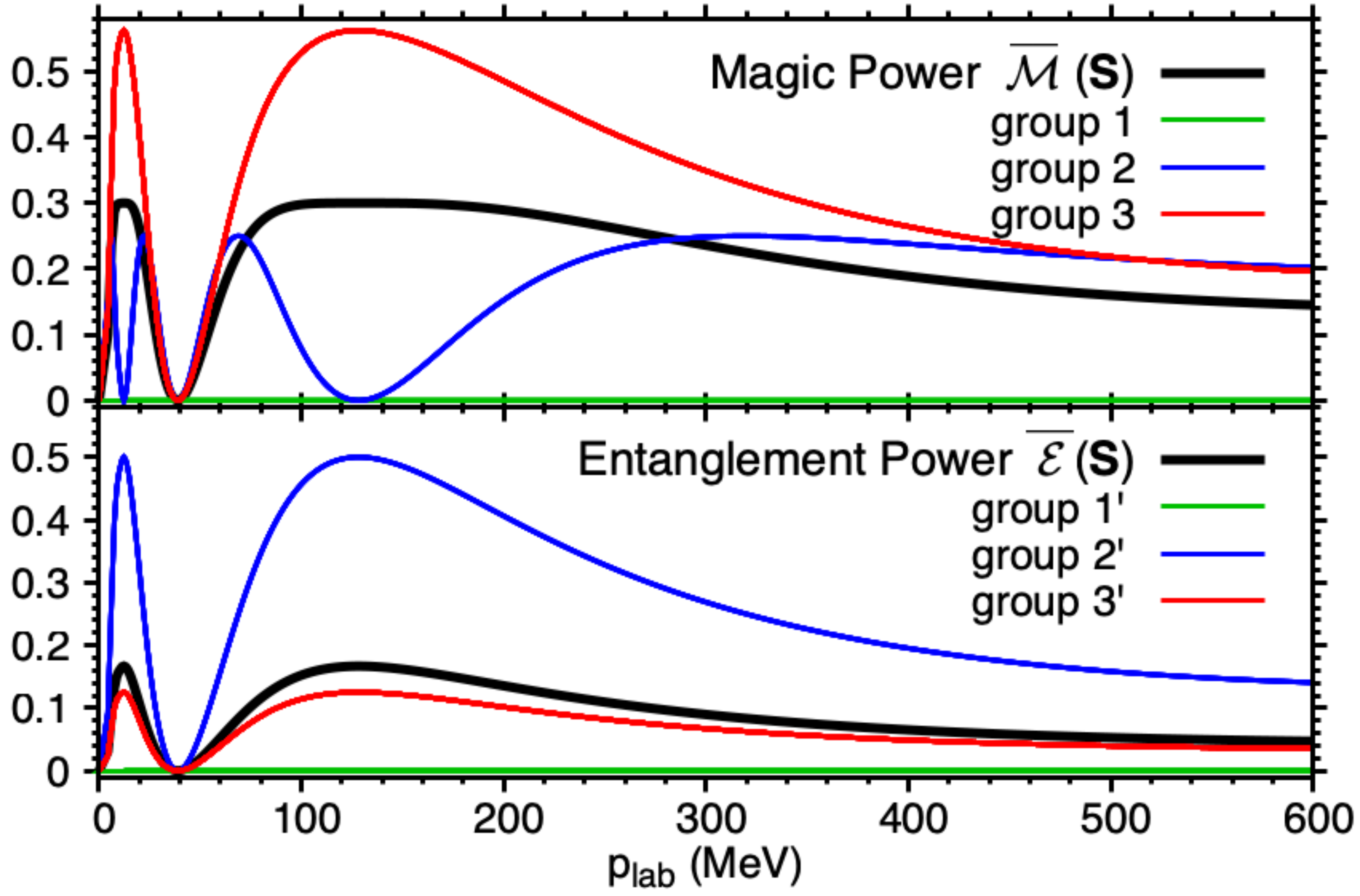


$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) = \frac{3}{20}\Big(3+\cos(4\,\Delta\delta)\Big)\sin^2(2\,\Delta\delta) \qquad \overline{\mathcal{E}}(\hat{\mathbf{S}}) = \frac{1}{6}\sin^2(2\,\Delta\delta)$$



Magic power of the S-matrix:

$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}}\sum_{i=1}^{\mathcal{N}_{ss}} \mathcal{M}\left(\hat{\mathbf{S}}\,|\Psi_i\rangle\right)$$

Average fluctuations in magic induced by the S-matrix

Entanglement power of the S-matrix

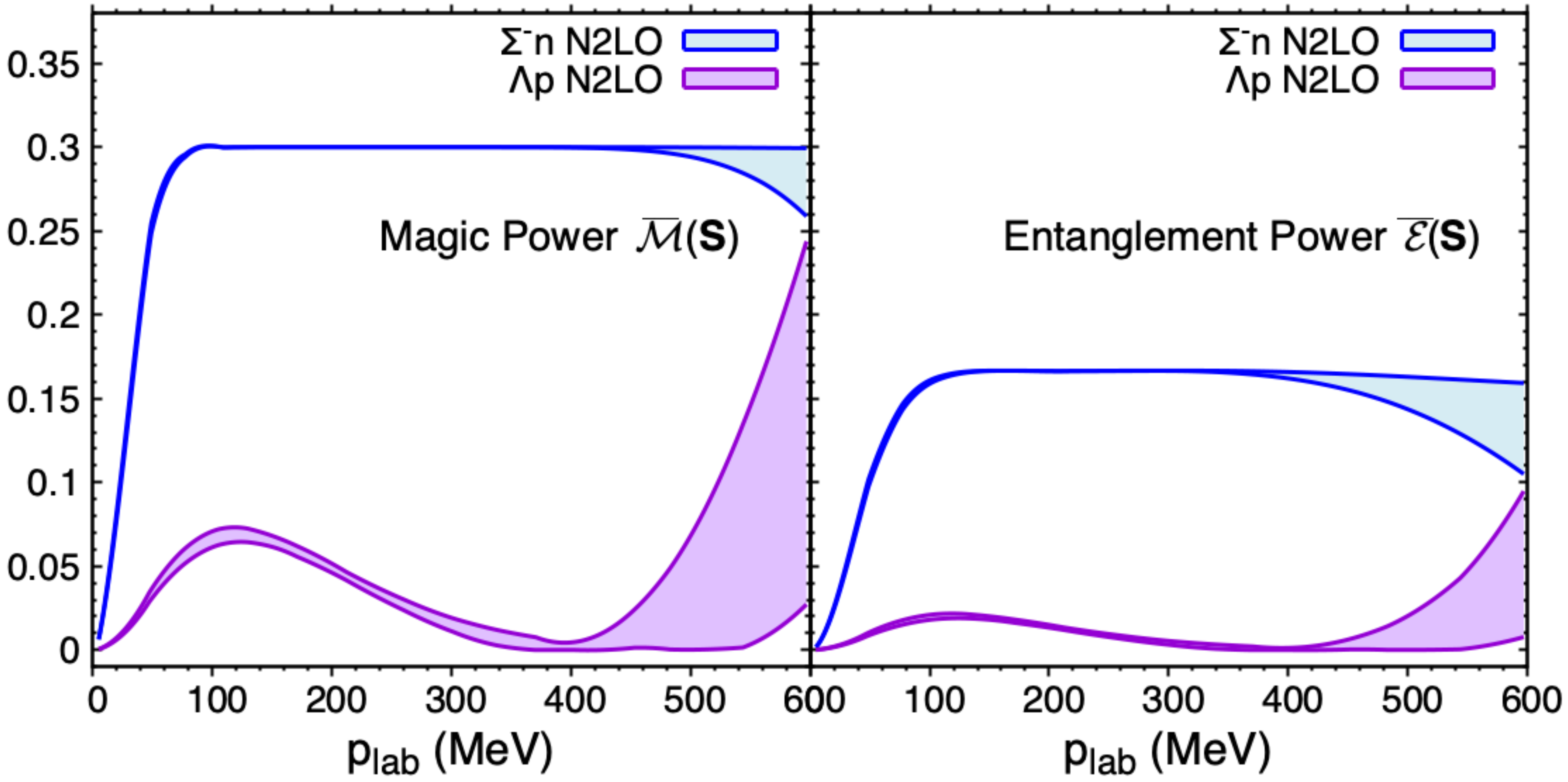
$$\overline{\mathcal{E}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}^{TP}}\sum_{i=1}^{\mathcal{N}_{ss}^{TP}} \mathcal{E}\left(\rho_i^{(1)}(\hat{\mathbf{S}})\right)$$

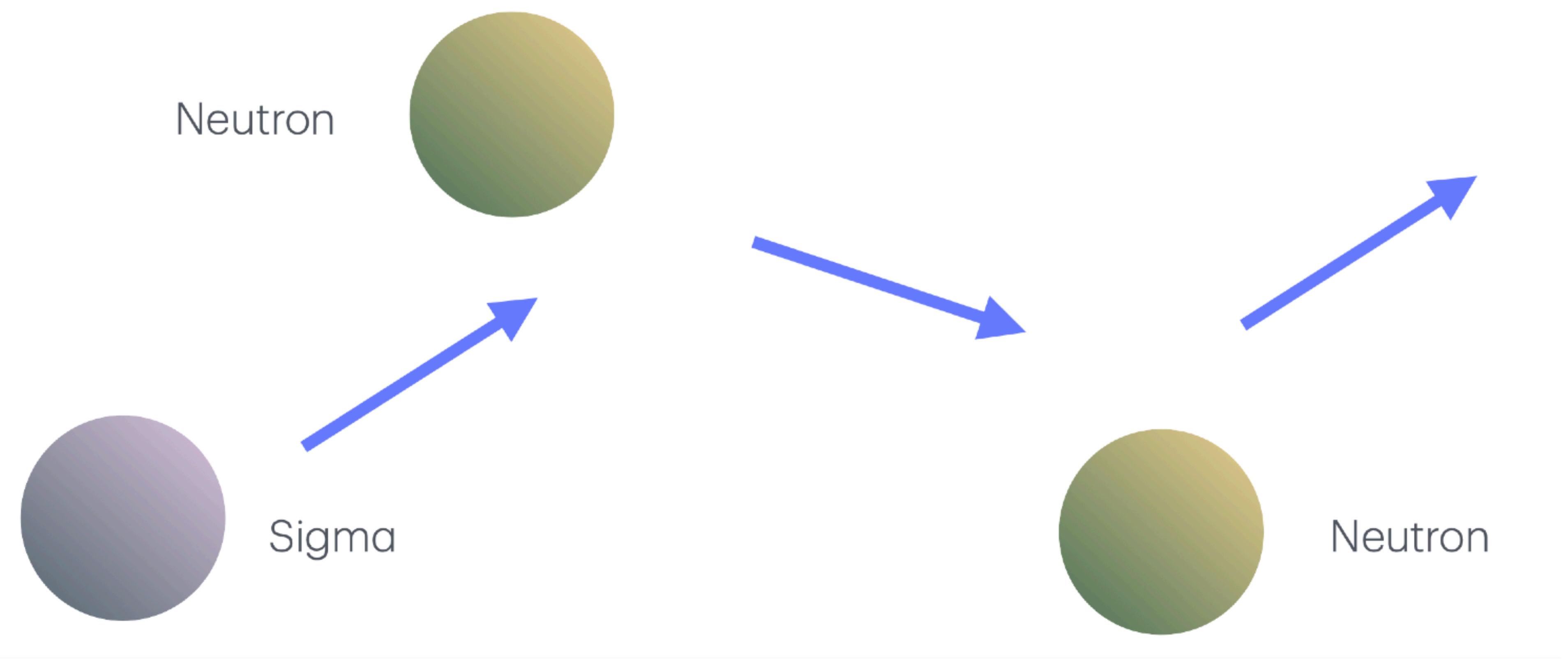
Same results as in Beane+ PRL 122, 102001 (2019) with continuous integration over spin orientations of initial tensor-product states

Magic power of a unitary operator (Leone, Oliviero, Hamma, PRL 128, 050402 (2022))

The Magic Power of Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268





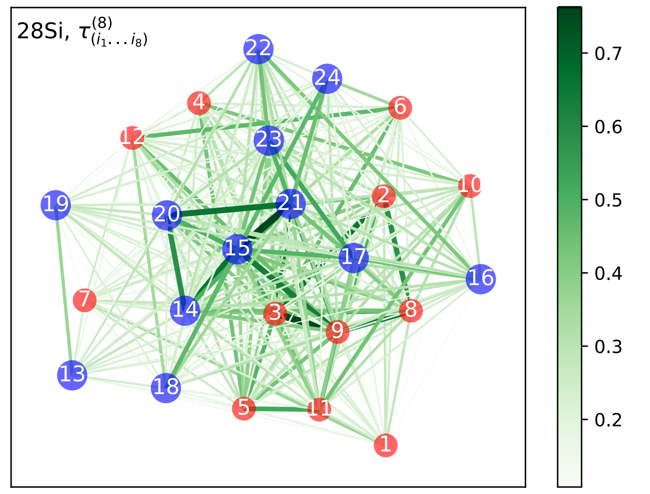
➡ Σ^--baryon is identified as a potential candidate catalyst for enhanced spreading of magic and entanglement in dense matter

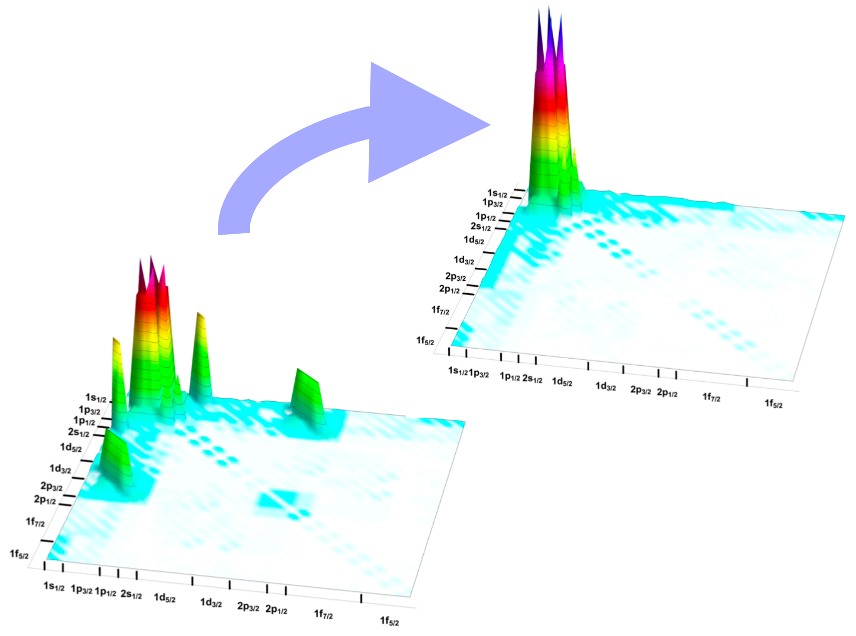
Entanglement power also in Beane+ PRL 122, 102001 (2019); Liu+ PLB 856, 138899 (2024)

Outline

★ Multi-Partite Entanglement and Non-Stabilizerness in Nuclei

Brökemeier, Hengstenberg, Keeble, CR, Rocco & Savage, arXiv:2409.12064



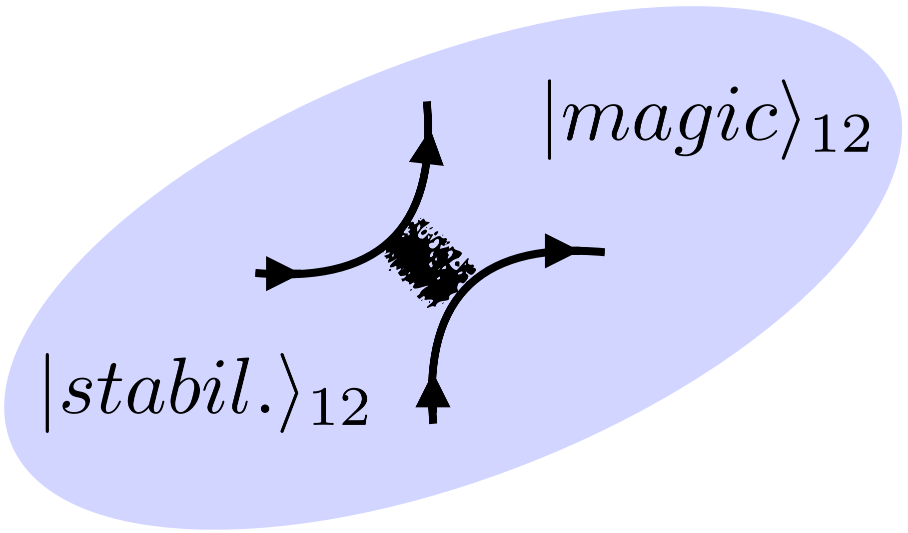


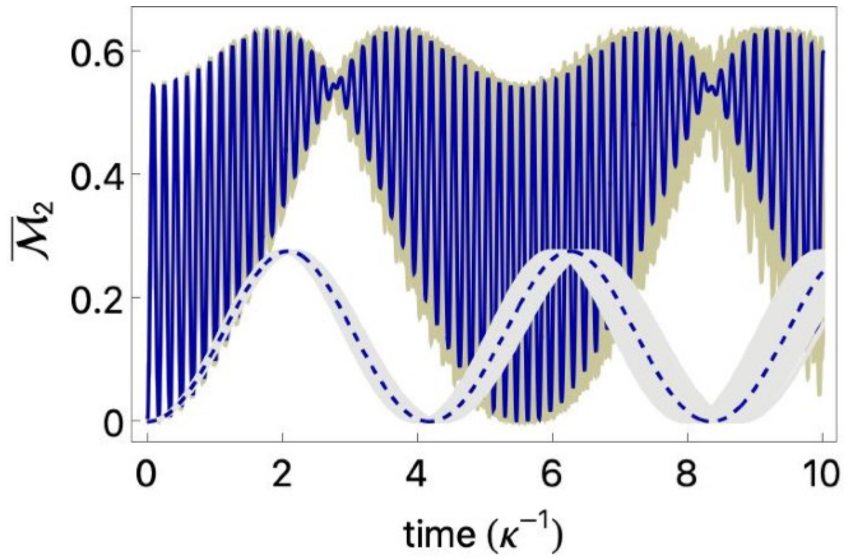
★ Entanglement and Magic Rearrangement for more efficient quantum simulations of QMB systems

CR, Savage, Pillet, PRC 103, 034325 (2021); CR & Savage PRC 108, 024313 (2023); Hengstenberg, CR, Savage EPJA 59, 231 (2023)

★ The Magic Power in Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268

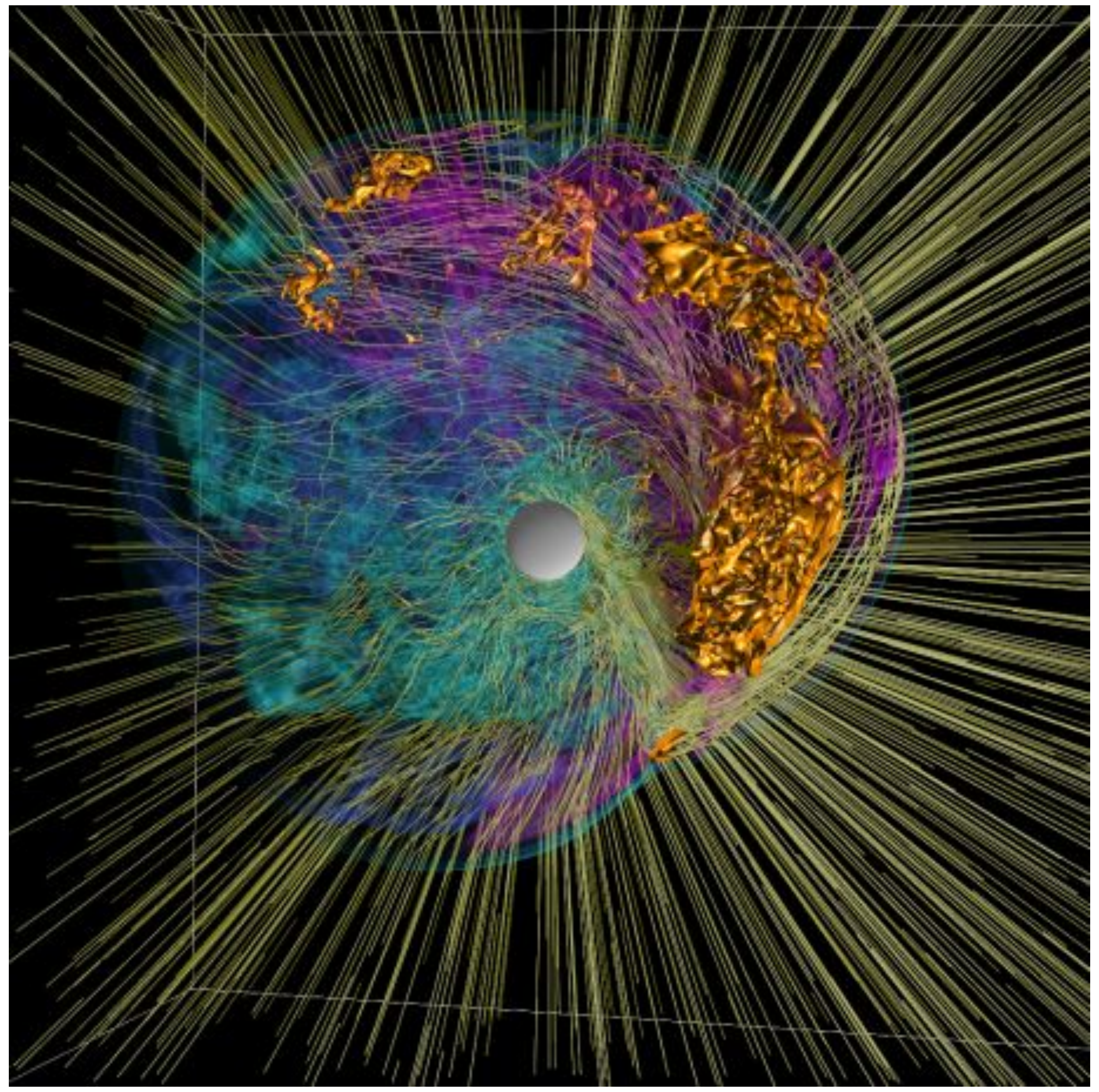




★ Entanglement and Magic in 3-flavour Neutrino dynamics mapped onto qutrits

Chernyshev, CR, Savage arXiv:2411.04203

Magic and Entanglement in Neutrino Dynamics



Neutrinos from core-collapse supernovae

$$\hat{H}_2(r) = \mu(r) \sum_{a=1}^{8} \hat{T}^a \otimes \hat{T}^a \ ,$$

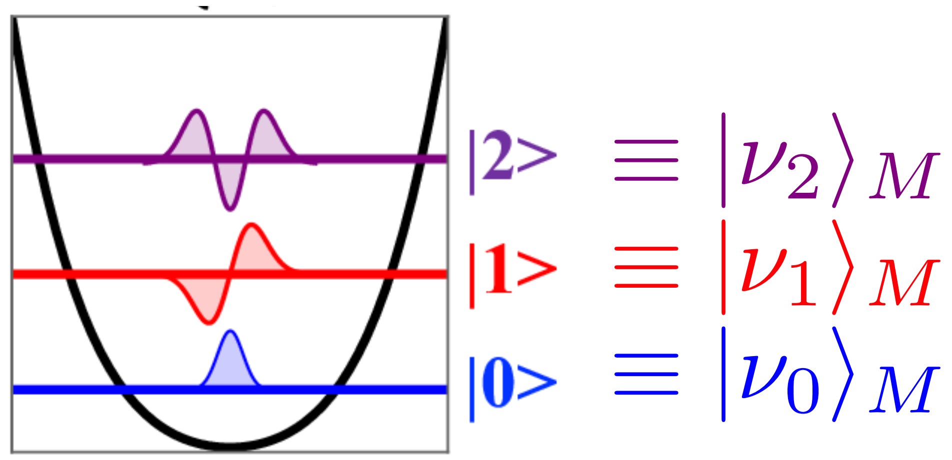
$$\mu(r) = \mu_0 \left(1 - \sqrt{1-(R_\nu/r)^2}\right)^2 \ ,$$

3 favours of neutrinos $|\nu\rangle_e, |\nu\rangle_\mu, |\nu\rangle_\tau$

Flavour to mass eigenstates: $|\nu\rangle_F = U_{PMNS}\,|\nu\rangle_M$

$$U_{PMNS} = \begin{pmatrix} 0.823300 & 0.547975 & -0.122396 + i0.083181 \\ -0.294674 + i0.051493 & 0.607002 + i0.034273 & 0.735451 \\ 0.480155 + i0.046295 & -0.573713 + i0.030813 & 0.661219 \end{pmatrix}$$

Mapping onto qutrits:



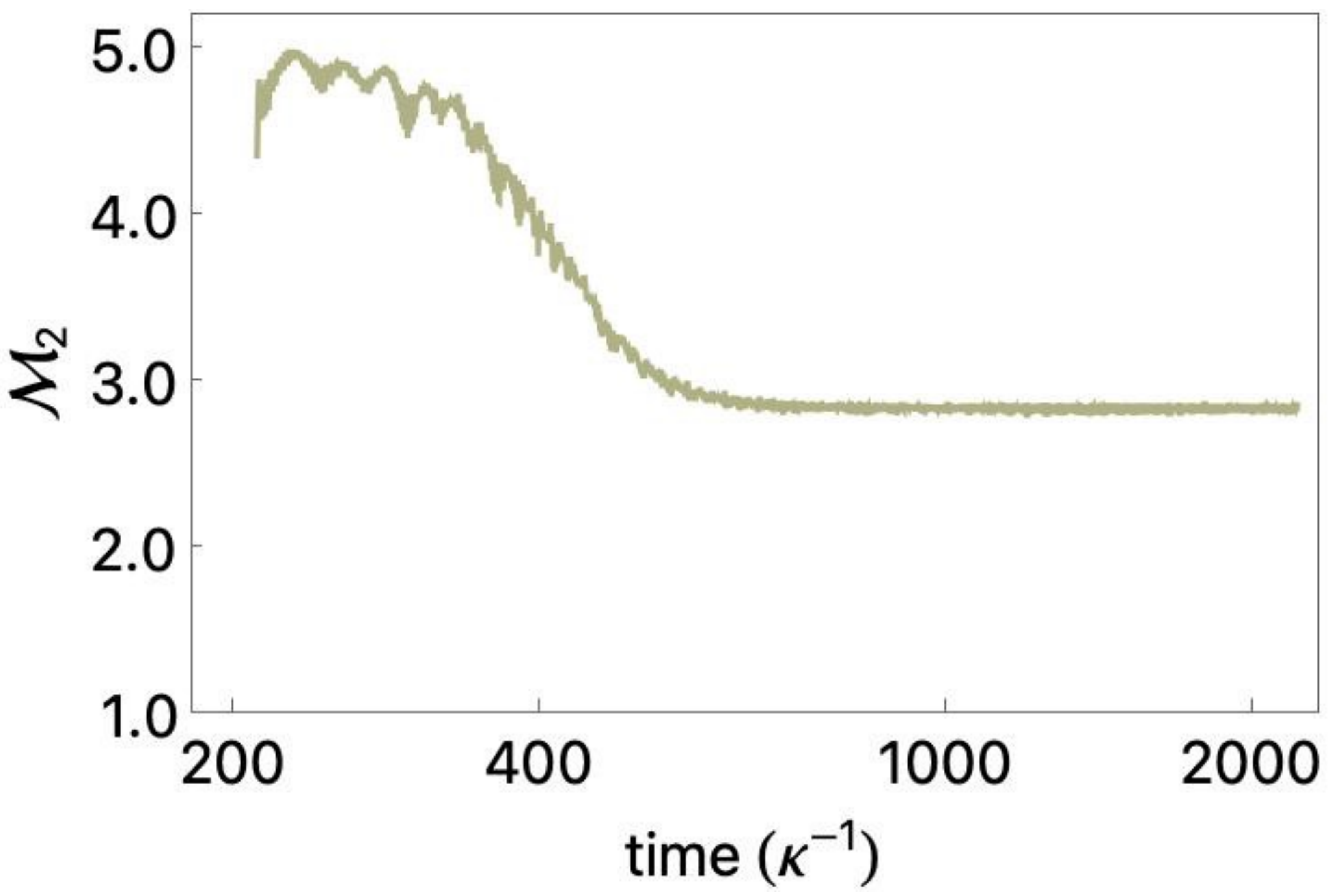
What is the evolution of entanglement and magic?

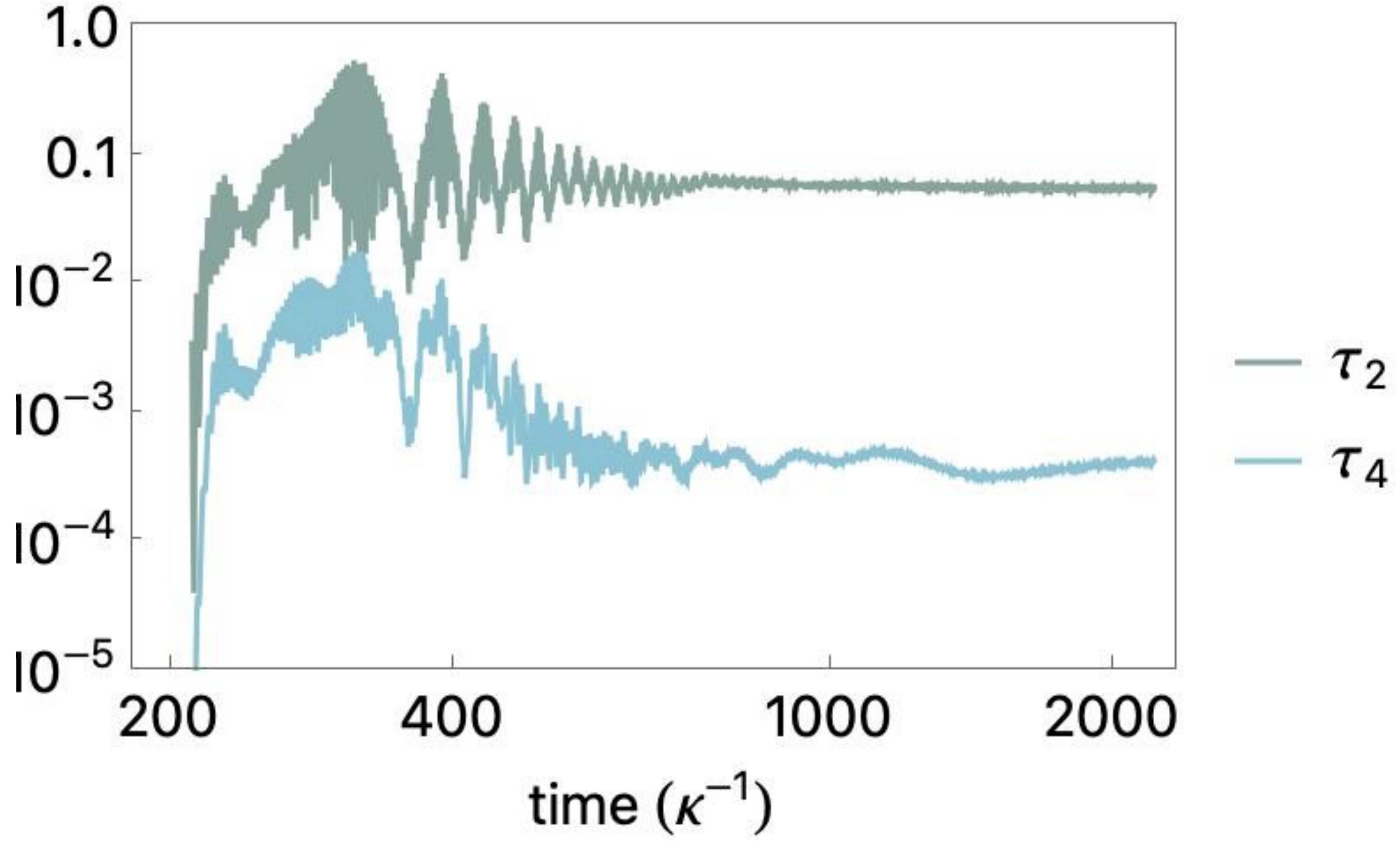
$$|\Psi(0)\rangle = |\nu\rangle_F^{(1)} \otimes ..|\nu\rangle_F^{(N)}$$

$$|\psi(t)\rangle = \hat{U}_2(t,0)|\psi\rangle_0 \ = \ T\left[e^{-i\int_0^t \ dt' \ \hat{H}(t')}\right]|\psi\rangle_0$$

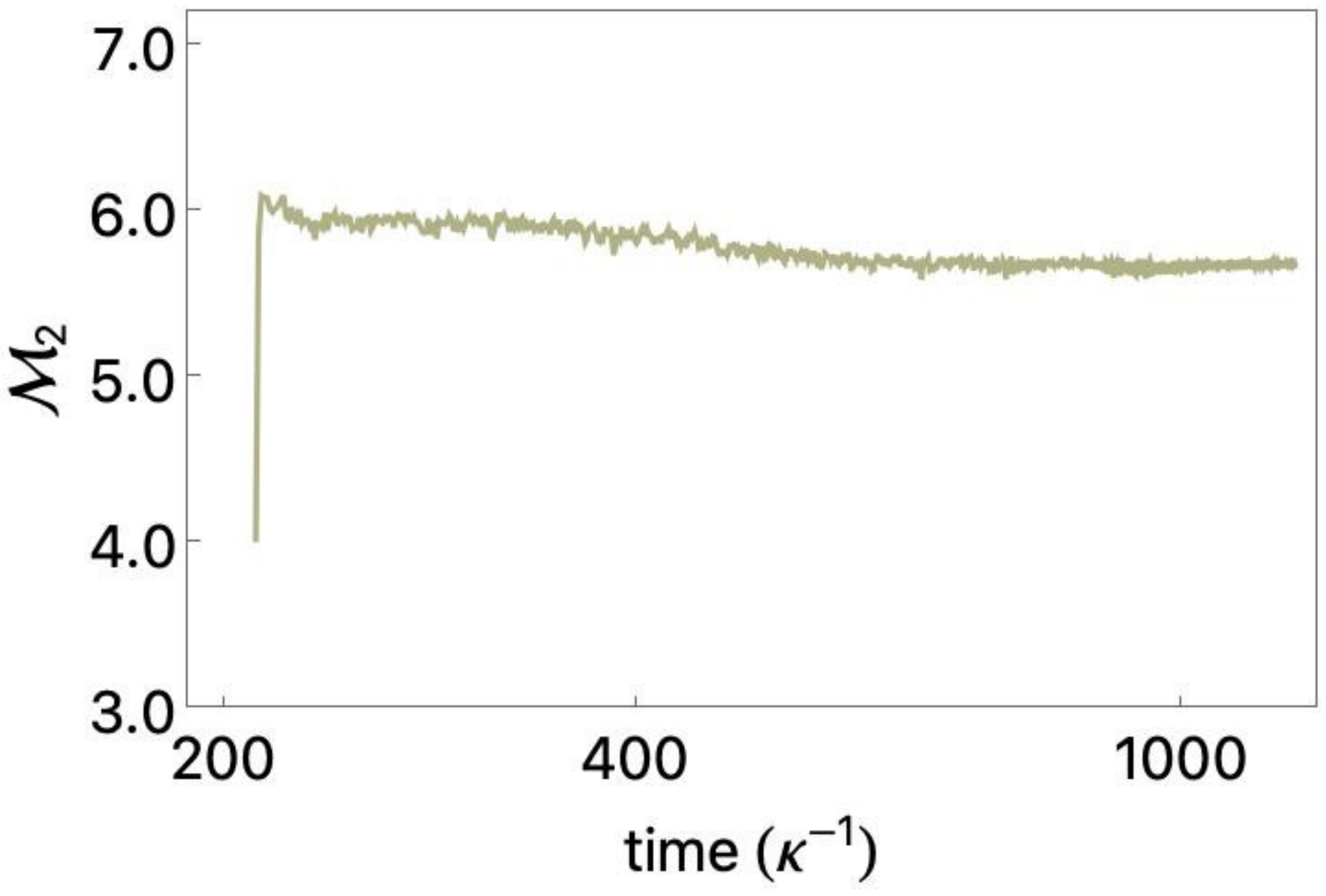
Magic and Entanglement in Neutrino Dynamics

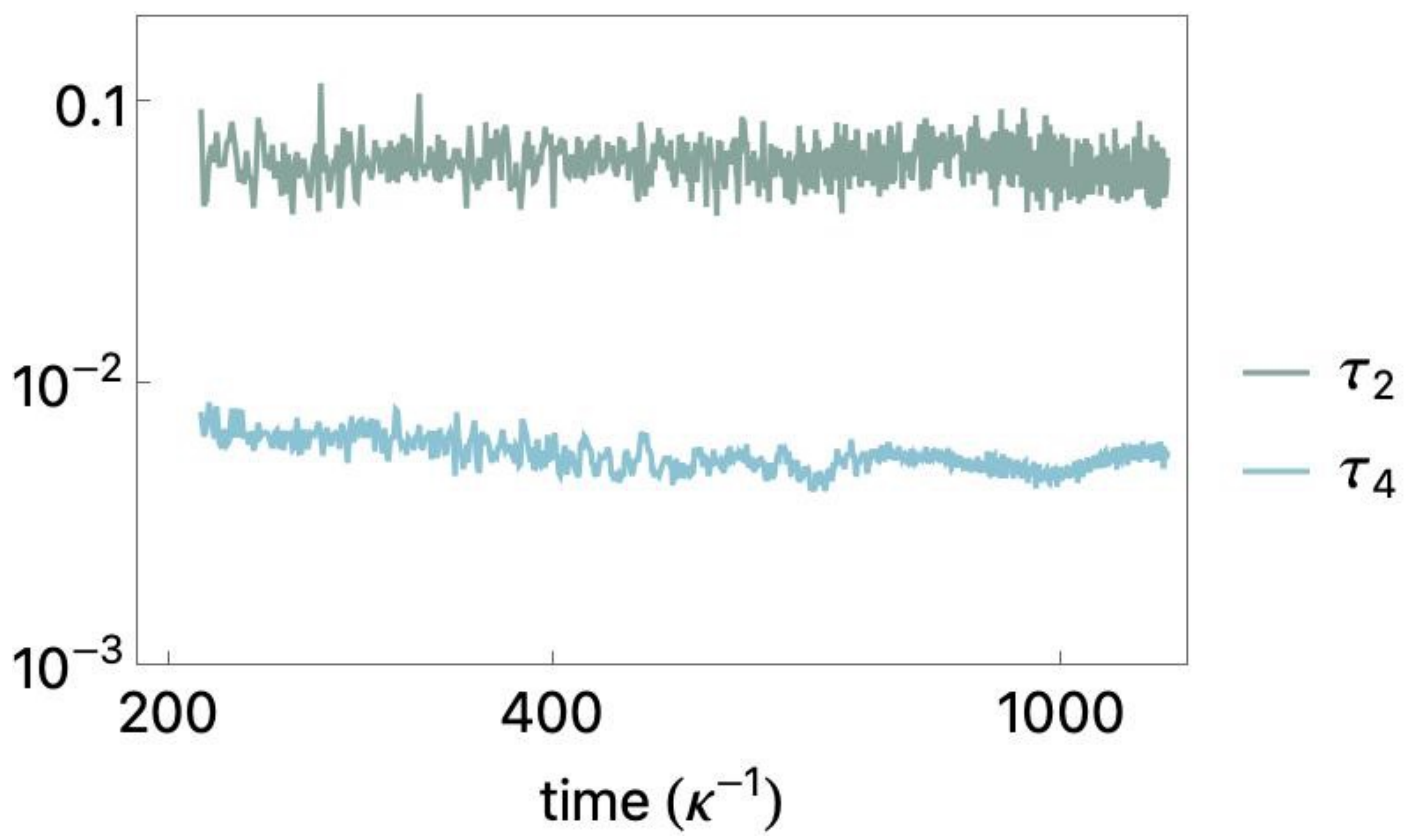
$|\nu_e\rangle^{\otimes 5}$





$|\nu_e\nu_e\nu_\mu\nu_\mu\nu_\tau\rangle$

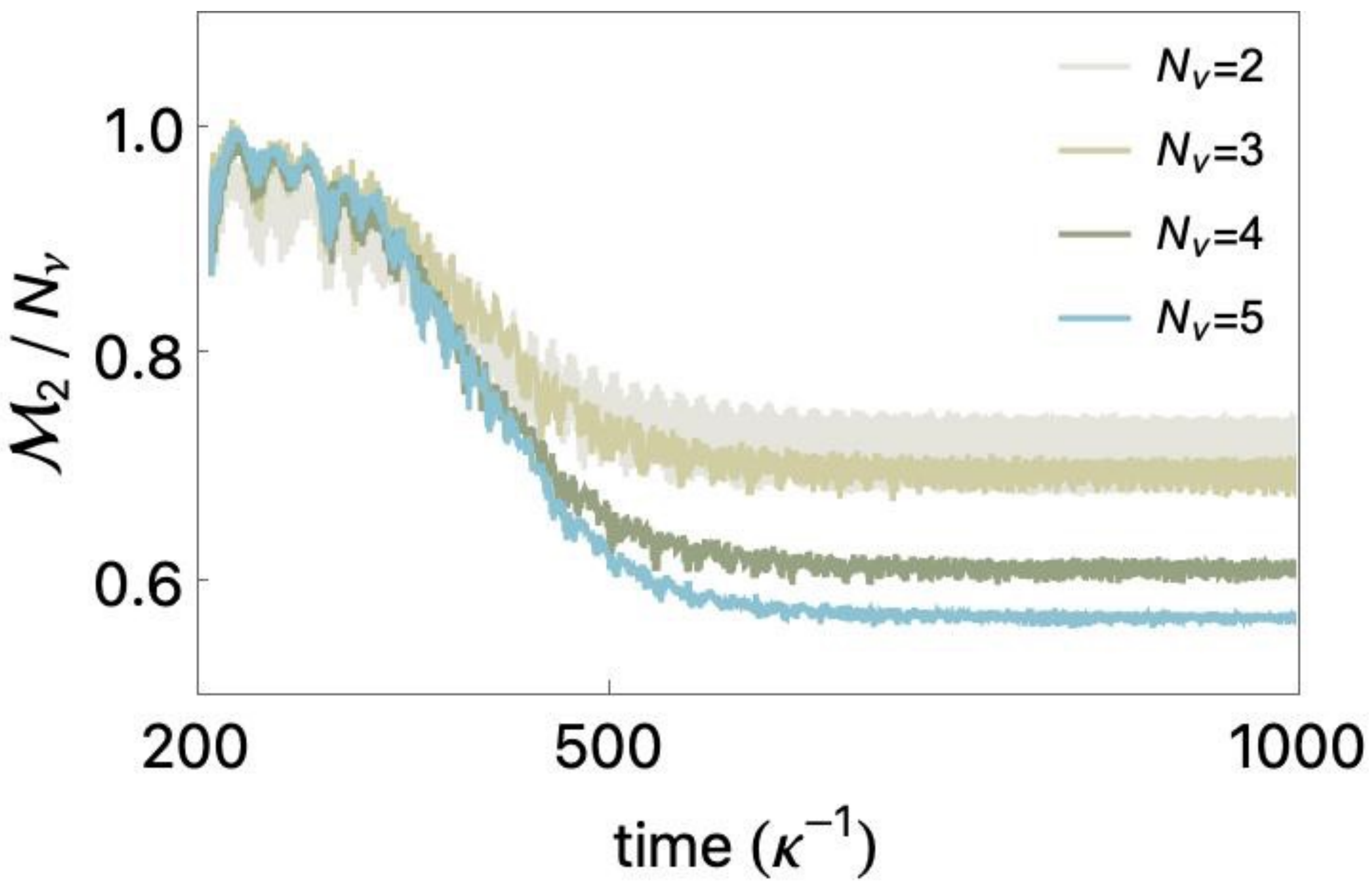




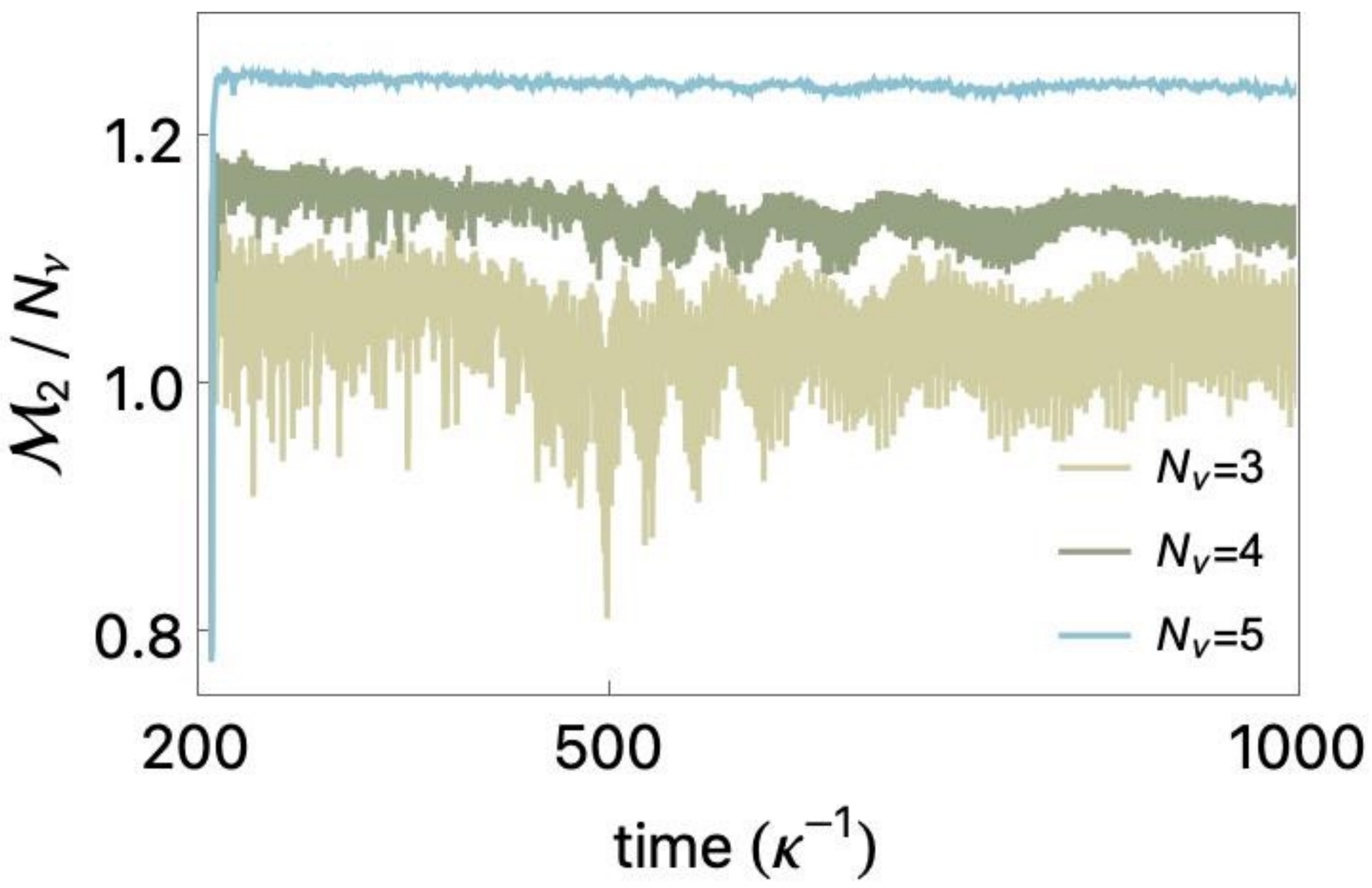
Chernyshev, CR, Savage arXiv:2411.04203

Magic and Entanglement in Neutrino Dynamics

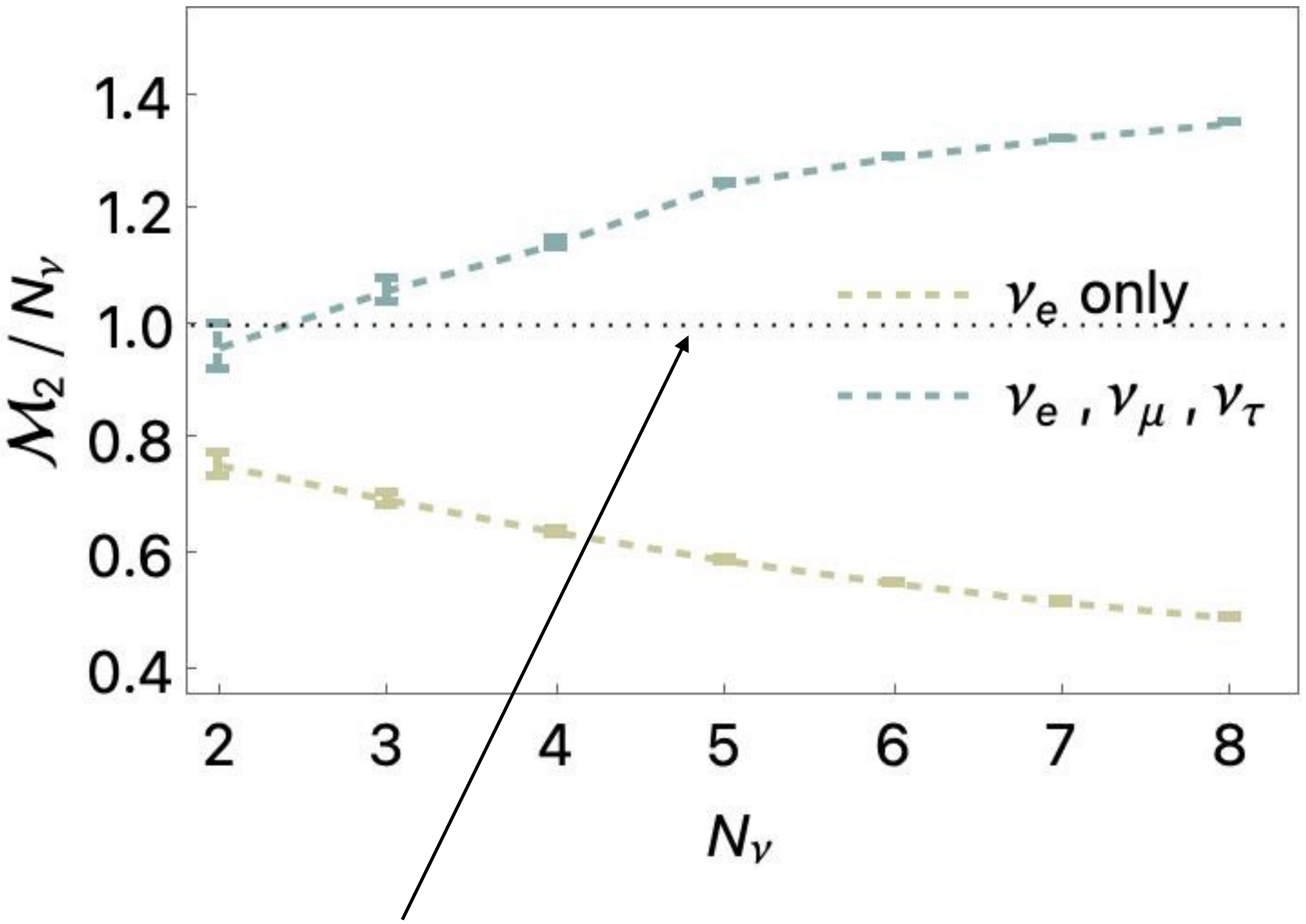
$|\nu_e\rangle^{\otimes N_\nu}$



Mixed flavours



Asymptotic values:



Max value for a tensor-product state

Chernyshev, CR, Savage arXiv:2411.04203

Conclusion

★ Entanglement and Magic provide new insights into various phenomena occurring in QMB systems

→ non-local magic in nuclear and neutrino systems?

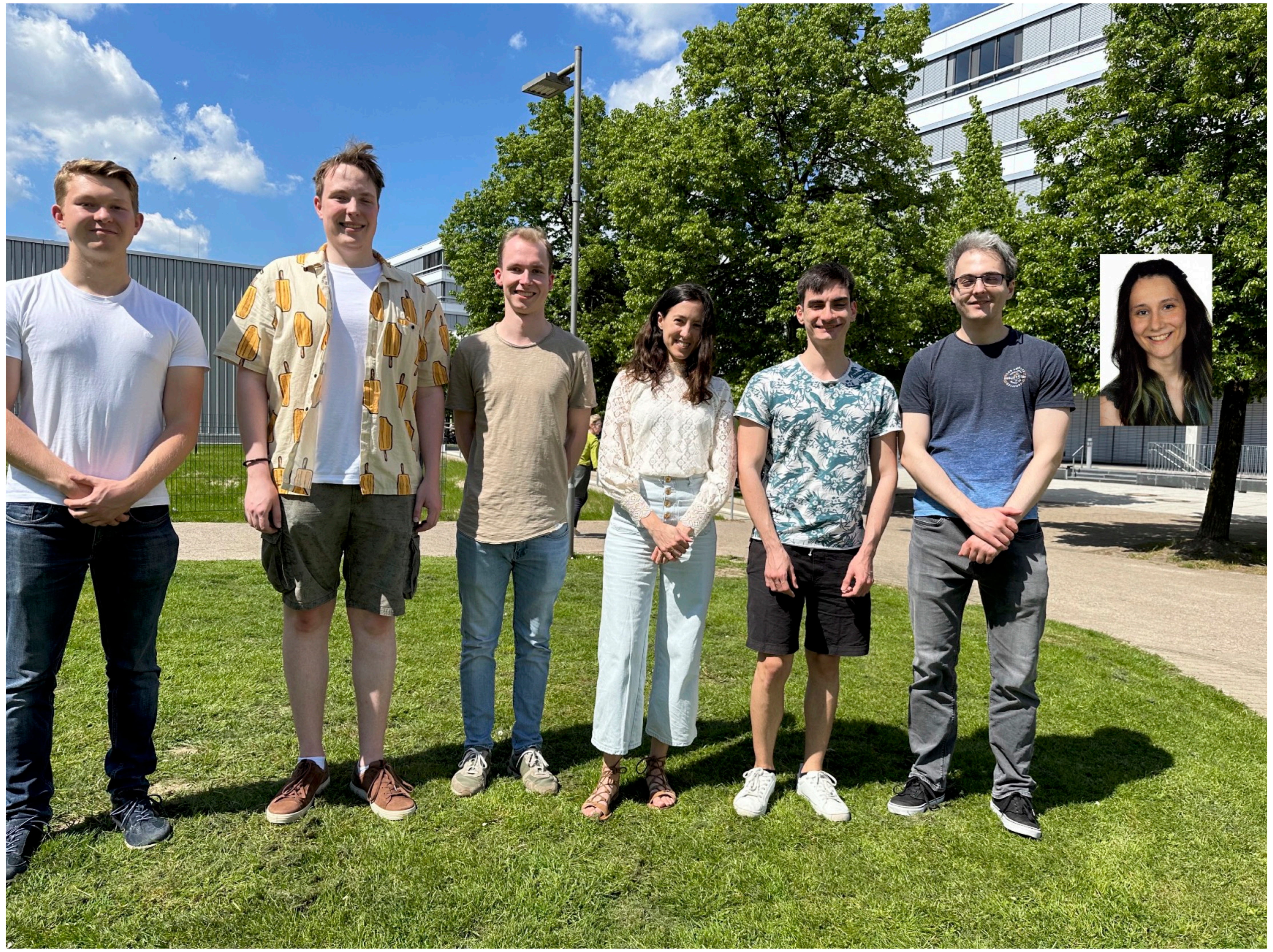
★ Entanglement, Magic and Symmetries are key ingredients for designing efficient hybrid classical/quantum simulations of structure and dynamics of QMB systems

→ developments on-going

★ Exchanges of ideas and techniques between fields of QMB physics and QIS is essential

Thanks to Collaborators!

Uni Bielefeld group



From left to right:
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Martin Savage

Marc Illa

Ivan Chernyshev